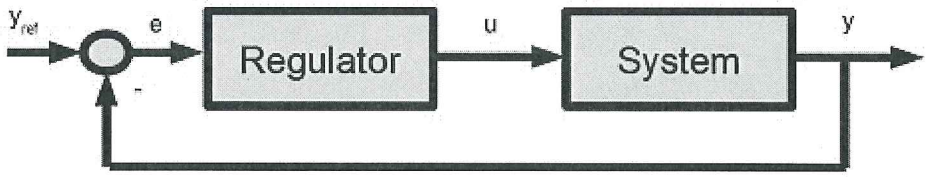
UNIVERSITETET I AGDER
FAKULTET FOR TEKNOLOGI OG REALFAG

E K S A M E N
Deleksamen 2/Final Exam 2

MAS107 Reguleringsteknikk



Dato: 9. desember 2011, Varighet/duration: 0900 - 1200

Tillatte hjelpemidler: Godkjent kalkulator og alle skriftlige hjelpemidler.
Permitted: Approved calculator and all written material
(PC/mobiltelefon ikke tillatt/No PC or mobile phone)

Eksamen består av 4 oppgaver med delspørsmål.
Alle oppgavene skal besvares.
The Exam consists of 4 problems with subproblems.
All problems shall be solved.
You can write your solutions in Norwegian or English.

Alle oppgaver i dette settet kan løses uavhengig av hverandre
All problems in this exam can be solved independently of each other.

To Bode-plot skal leveres som en del av besvarelsen, i to eksemplarer.
Husk å merke Bode-plot med oppgavenummer
samt ditt eget kandidatnummer.

Two Bode plots shall be handed in as part of the solution, in two copies.
Remember: Mark plots with problem number
as well as your own candidate number

Tabeller med Laplace-transformasjoner er vedlagt. **Må** brukes i oppgave 3.
Laplace transform tables are provided. **Must be** used in problem 3.

Function Transforms		
N	F(s)	f(t) , t > 0
2.1	1	$\delta(t)$, unit impulse at t = 0
2.2	s	$\frac{d}{dt}\delta(t)$, doublet impulse at t = 0
2.3	$\exp(-\alpha s)$, $\alpha \geq 0$	$\delta(t-\alpha)$
2.4	$\frac{1}{s}$	u(t) , unit step
2.5	$\frac{1}{s}\exp(-\alpha s)$	$u(t-\alpha)$
2.6	$\frac{1}{s^2}$	t
2.7	$\frac{1}{s^n}$, n = 1, 2, 3, ...	$\frac{t^{n-1}}{(n-1)!}$
2.8	$\frac{n!}{s^{n+1}}$, n = 1, 2, 3, ...	t^n
2.9	$\frac{1}{s^k}$, k is any real number > 0	$\frac{t^{k-1}}{\Gamma(k)}$, the Gamma function is given in Appendix A
2.10	$\frac{1}{s+\alpha}$	$\exp(-\alpha t)$
2.11	$\frac{1}{(s+\alpha)^2}$	$t\exp(-\alpha t)$

Function Transforms		
N	F(s)	f(t) , t > 0
2.12	$\frac{1}{(s+\alpha)^n}$, n = 1, 2, 3, ...	$\left[\frac{t^{n-1}}{(n-1)!}\right]\exp(-\alpha t)$
2.13	$\frac{\alpha}{s(s+\alpha)}$	$1-\exp(-\alpha t)$
2.14	$\frac{1}{(s+\alpha)(s+\beta)}$, $\alpha \neq \beta$	$\frac{1}{(\beta-\alpha)}\left[\exp(-\alpha t)-\exp(-\beta t)\right]$
2.15	$\frac{1}{s(s+\alpha)(s+\beta)}$, $\alpha \neq \beta$	$\frac{1}{\alpha\beta}+\frac{\exp(-\alpha t)}{\alpha(\alpha-\beta)}+\frac{\exp(-\beta t)}{\beta(\beta-\alpha)}$
2.16	$\frac{s}{(s+\alpha)(s+\beta)}$, $\alpha \neq \beta$	$\frac{1}{(\alpha-\beta)}\left[\alpha\exp(-\alpha t)-\beta\exp(-\beta t)\right]$
2.17	$\frac{s}{(s+\alpha)^2}$	$\left[1-\alpha t\right]\exp(-\alpha t)$
2.18	$\frac{1}{s(s+\alpha)^2}$	$\frac{1}{\alpha^2}\left\{1-\left[1+\alpha t\right]\exp(-\alpha t)\right\}$
2.19	$\frac{\alpha}{s^2+\alpha^2}$	$\sin(\alpha t)$
2.20	$\frac{\left[\sin(\phi)\right]s+\left[\cos(\phi)\right]\alpha}{s^2+\alpha^2}$	$\sin(\alpha t+\phi)$
2.21	$\frac{s}{s^2+\alpha^2}$	$\cos(\alpha t)$
2.22	$\frac{s^2-\alpha^2}{\left(s^2+\alpha^2\right)^2}$	$t\cos(\alpha t)$
2.23	$\frac{1}{s\left(s^2+\alpha^2\right)}$	$\frac{1}{\alpha^2}\left[1-\cos(\alpha t)\right]$

Oppgave 1) Bode-analyse / Problem 1) Bode analysis (25 %)

Table 1: Frequency response for system G(s)

Omega	Amp (dB)	Phase
0,1	19,956352	-96,2835318
0,20892961	13,4125493	-102,997933
0,30199517	10,0169931	-108,533842
0,39810717	7,354202	-113,987638
0,50118723	5,01587676	-119,488635
0,60255959	3,03905302	-124,519711
0,69183097	1,48066754	-128,634279
0,79432823	-0,15174207	-133,002738
0,91201084	-1,86466829	-137,576176
1	-3,05351369	-140,710593
1,20226443	-5,545895	-147,103106
1,38038426	-7,51412465	-151,938428
1,58489319	-9,56314689	-156,755684
1,81970086	-11,6875725	-161,522678
1,99526231	-13,1427708	-164,664385
3,01995172	-20,0308995	-178,482748
3,98107171	-24,9046441	-187,607533
6,02559586	-32,6632043	-201,648618
7,94328235	-38,1927173	-211,285745
10	-43,0535137	-219,289407
20,8929613	-60,1057632	-241,682563
30,1995172	-69,256611	-249,782137
50,1187234	-82,1712715	-257,573121
69,1830971	-90,4907094	-260,947086
100	-100,043648	-263,716468
301,995172	-128,804807	-267,913728
501,187234	-142,001746	-268,742631
1000	-160,000439	-269,369766

Tabell 1 ovenfor viser tallverdier for frekvensresponsen til et system G(s).

a) Lag et komplett Bode-plott med amplitude og faserespons for G(s). Bruk og tilpass et av de tomme bodeplottene.

b) Bestem forsterkningsmarginen og fasemarginen for system G(s), basert på Tabell 1 eller ditt eget Bode-plott.

c) Estimer transferfunksjonen til system G(s). (Hvilken transferfunksjon ligger mest sannsynlig til grunn for responsen i Tabell 1?)

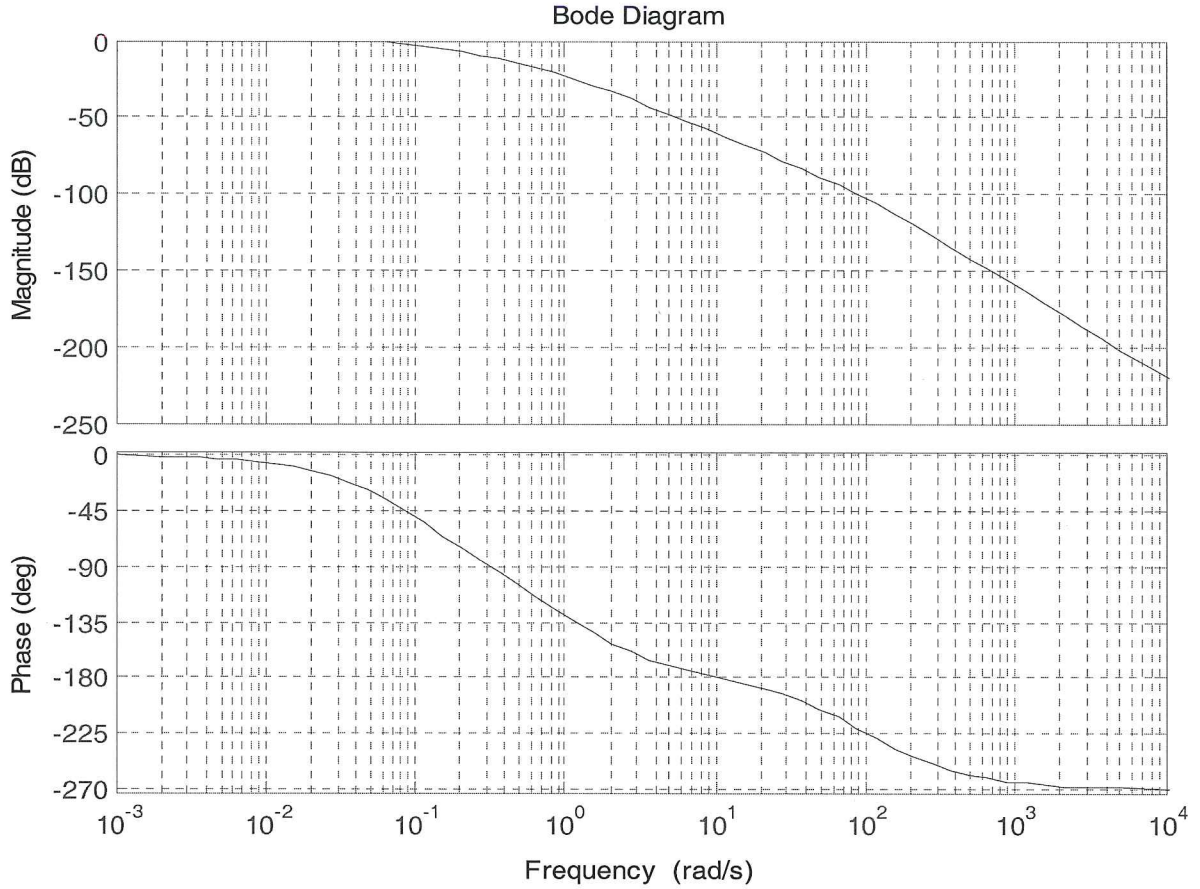
Table 1 above shows numerical values for the frequency response of a system G(s).

a) Provide a complete Bode plot with amplitude and phase response for G(s). Use and adapt one of the empty Bode plots.

b) Based on Table 1 or your own Bode plot, determine the gain margin and the phase margin for system G(s).

c) Estimate the transfer function for system G(s). (What is the most probable transfer function to give the response in Table 1?)

Oppgave 2) Lag kompensator /Problem 2) Lag compensator (25 %)



Figur 1: Bodeplott for system $G_1(s)$/Bode plot for system $G_1(s)$
Amplitude skala 5 dB per millimeter/Amplitude scale 5 dB per millimeter

Figur 1 ovenfor viser et Bode plott med amplitude og fase respons for systemet $G_1(s)$. Vi benytter så dette systemet i en enhets tilbakekopling.

a) Hvilken fase og amplitudemargin har det tilbakekoplede systemet dersom vi benytter en P-regulator med forsterkning lik 30?

b) Hva blir de stasjonære avvik for et enhetssprang og for en enhetsrampe for systemet dersom vi benytter en P-regulator med forsterkning lik 30?

c) Vi går nå videre uten P-regulatoren, dvs P = 1. Vi ønsker nå en lag-kompensator for systemet $G_1(s)$, slik at stasjonært avvik ved en sprangrespons blir 1 % og oversving blir 35 %, (fortsatt i en enhets tilbakekopling). Vis i detalj at en rimelig kompensator kan være

$$G_{lag}(s) = \frac{100}{316} \cdot \frac{s+0.15}{s+0.0005}$$

Figure 1 above shows a Bode plot with amplitude and phase response for the system $G_1(s)$. Now we use this system in a unity feedback configuration.

a) What are the phase and amplitude margins for the feedback system if we use a P-controller with a gain equal to 30?

b) What are the stationary errors for a step input and for a ramp input for the system if we use a P-controller with gain equal to 30?

c) Now we proceed without the P-controller, this means P = 1. We want a lag compensator for the system $G_1(s)$ such that the stationary error for a step input becomes 1 % and the overshoot becomes 35 %, (still in a unity feedback configuration). Show in detail that a reasonable compensator can be

$$G_{lag}(s) = \frac{100}{316} \cdot \frac{s+0.15}{s+0.0005}$$

Oppgave 3) Systemanalyse og forståelse / Problem 3) System analysis and understanding (25 %)

a) Gitt overføringsfunksjonen $G_o(s)$ lik

$$G_o(s) = \frac{K}{s(s+4)}$$

Vi benytter $G_o(s)$ i et enhets tilbakekoplet system, der $G_{cl}(s)$ er overføringsfunksjonen for det tilbakekoplete systemet. Skriv ned formelen for det tilbakekoplete systemet $G_{cl}(s)$.

a) Given the transfer function $G_o(s)$ equal to

$$G_o(s) = \frac{K}{s(s+4)}$$

We use $G_o(s)$ in a unity feedback system, where the closed loop transfer function is called $G_{cl}(s)$. Write down the formula for the feedback system $G_{cl}(s)$.

b) La K = 4, skriv ned den resulterende transferfunksjonen $G_{cl}(s)$ og vis at den kan skrives på formen

$$G_{cl}(s) = \frac{a}{(s+a)} \bullet \frac{a}{(s+a)}$$

der du også bestemmer verdien for a.

b) Let K = 4, write down the resulting transfer function $G_{cl}(s)$ and show that it can be written on the form

$$G_{cl}(s) = \frac{a}{(s+a)} \bullet \frac{a}{(s+a)}$$

where you also determine the value for a.

c) Hva er polene til $G_{cl}(s)$? Lag en skisse som viser poler og nullpunkter for $G_{cl}(s)$ i s-planet.

c) What are the poles of $G_{cl}(s)$? Draw a sketch showing poles and zeroes in the s-plane for $G_{cl}(s)$.

d) Bestem ved hjelp av Laplace-transformasjoner trinnpulsresponsen (tidsresponsen) for $G_{cl}(s)$. (Du kan bruke en vilkårlig verdi for a om du ikke finner den riktige).

d) Using Laplace transforms, determine the step response (time response) for $G_{cl}(s)$. (You can use an arbitrary value for a if you cannot find the right one).

e) Bonusspørsmål. **Løs bare hvis du har tid.** Bruk kurver fra læreboka der hvor du ønsker. Sett K = 40. Hva blir dempningsfaktoren ζ, og hva blir % oversving i trinnpulsresponsen? Vi betrakter også den normaliserte amplituderesponsen for $G_{cl}(s)$, (som har 0 dB forsterkning ved null frekvens). Den har en toppverdi. Hvilken forsterkning i dB har amplitudetoppen?

e) Bonus question. **Solve only if time allows.** Use curves from the textbook whenever suitable. Set K = 40. What is now the damping factor ζ and what is the % overshoot in the step response? We also consider the normalized amplitude response for $G_{cl}(s)$, (which has 0 dB gain at zero frequency). It has a peak value. What gain in dB has the amplitude peak?

Oppgave 4) Lead kompensator / Problem 4) Lead compensator (25 %)

Vi arbeider nå videre med prosessen $G_1(s)$ fra oppgave 2.

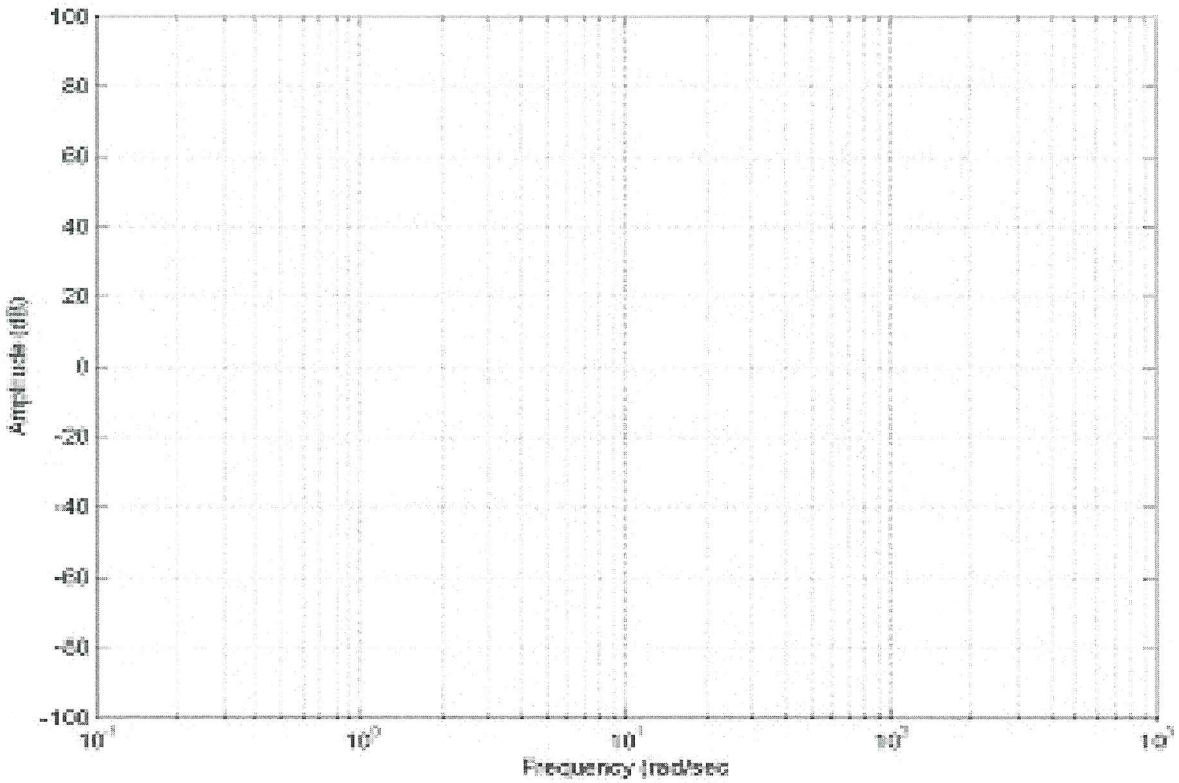
We continue working with the process $G_1(s)$ from problem 2.

a) Design en lead kompensator for systemet med frekvensrespons som vist i oppgave 2 slik at stasjonært avvik ved et enhetssprang blir lik 1 % og med oversving lik 30 %. Skriv ned transferfunksjonen til leadkompensatoren alene og leadkompensatoren inkludert den nødvendige forsterkningen for å oppnå ønsket nøyaktighet. (Tips: Bruk målestokken til Figur 1 til å estimere mest mulig nøyaktige amplituderesponser.)

a) Design a lead compensator for the system with a frequency response as shown in problem 2, such that the stationary error with a unity step function becomes 1 % and with an overshoot of 30 %. Write down the transfer functions for the lead compensator alone, and the lead compensator included the gain required for achieving the wanted accuracy. (Hint: Use the scale of Figure 1 to estimate amplitude responses as accurately as possible).

b) Tegn asymptotisk Bode-plot i tillegg til en mer nøyaktig tilnærming av frekvensresponsen til lead kompensatoren alene, med både amplitude og fase. (Dvs. forsterkningen for å oppnå ønsket stasjonært avvik skal ikke regnes eller tegnes inn). Bruk et av de vedlagte tomme Bode-plottene. Tilpass skalering i frekvens eller amplitude der hvor det måtte være nødvendig.

b) Draw an asymptotic Bode plot as well as a more accurate approximation of the lead compensator frequency response in amplitude as well as phase, for the lead compensator alone (This means that the gain necessary for achieving the wanted stationary error shall not be counted or drawn in). Use one of the attached empty Bode plots. Adapt frequency or amplitude scaling whenever necessary.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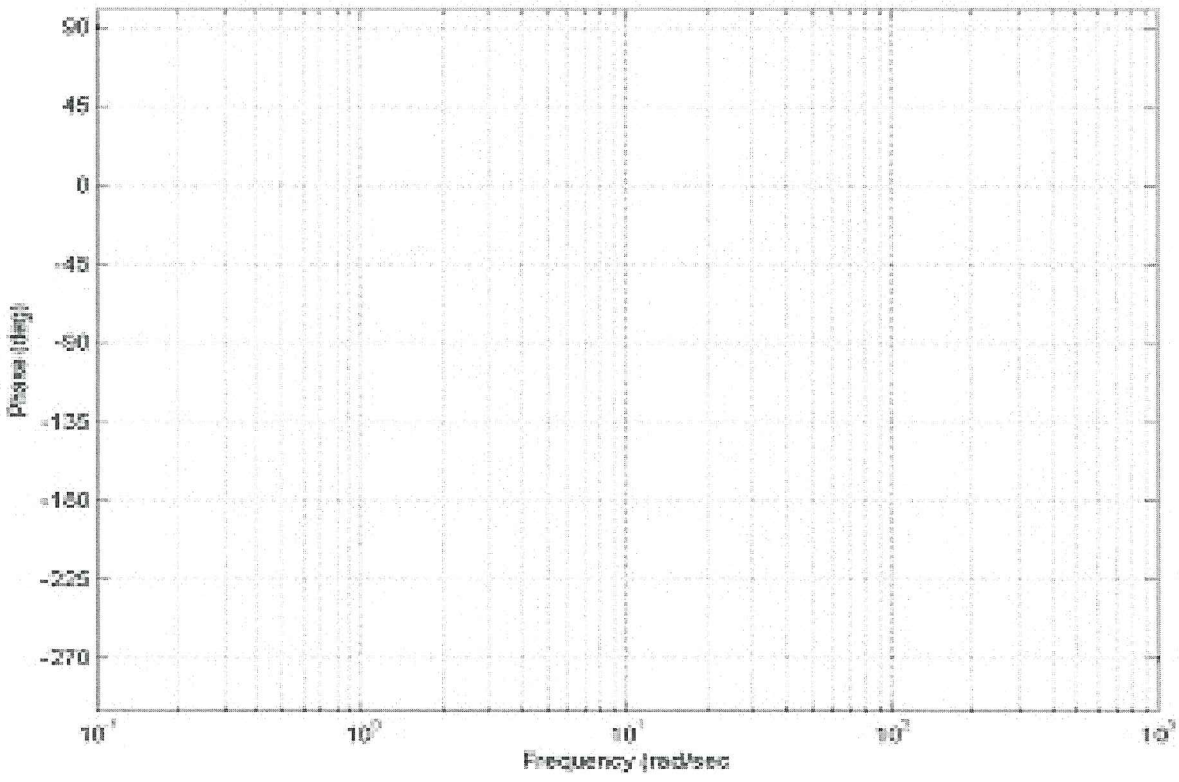
Frequency (rad/sec)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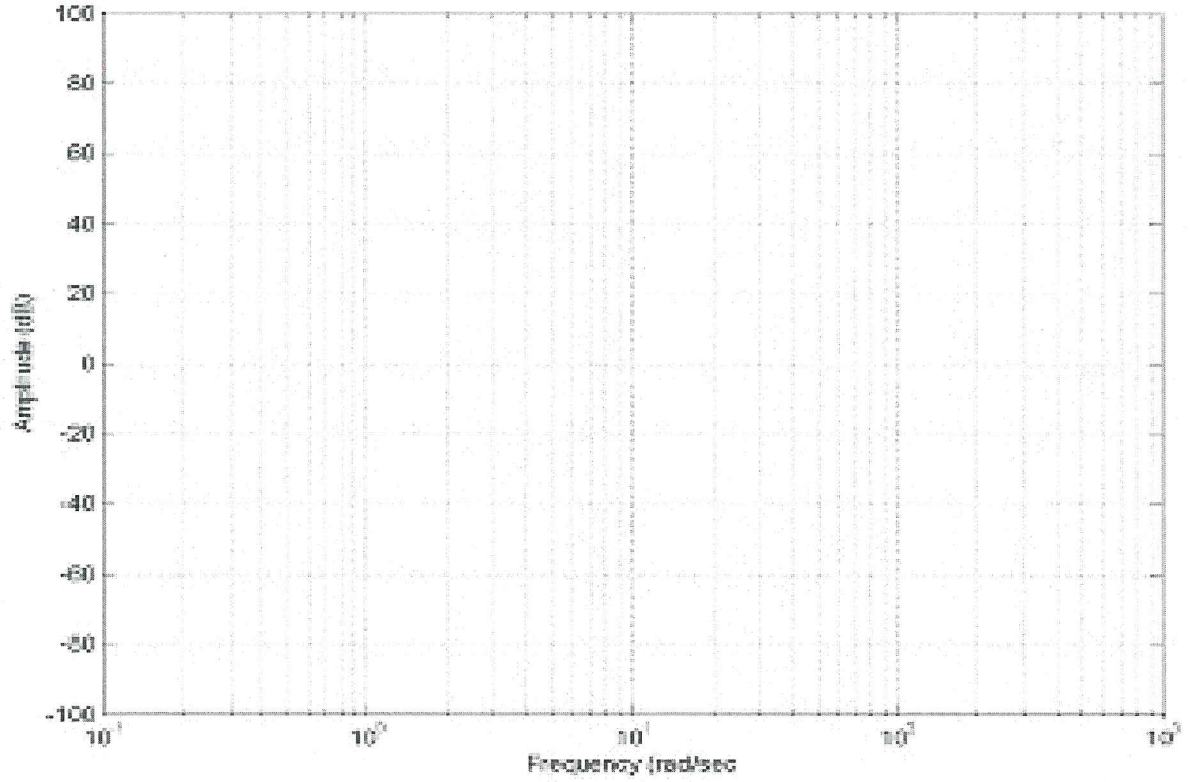
Frequency (rad/sec)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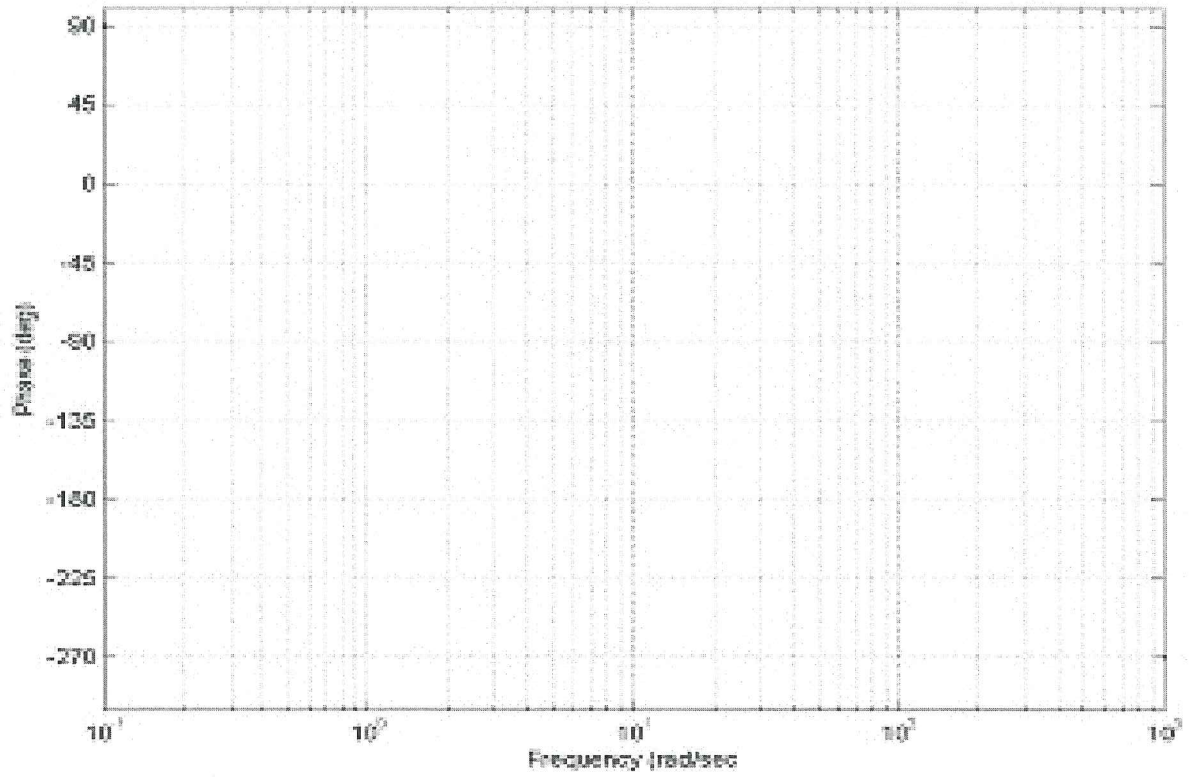
Frequency (rad/sec)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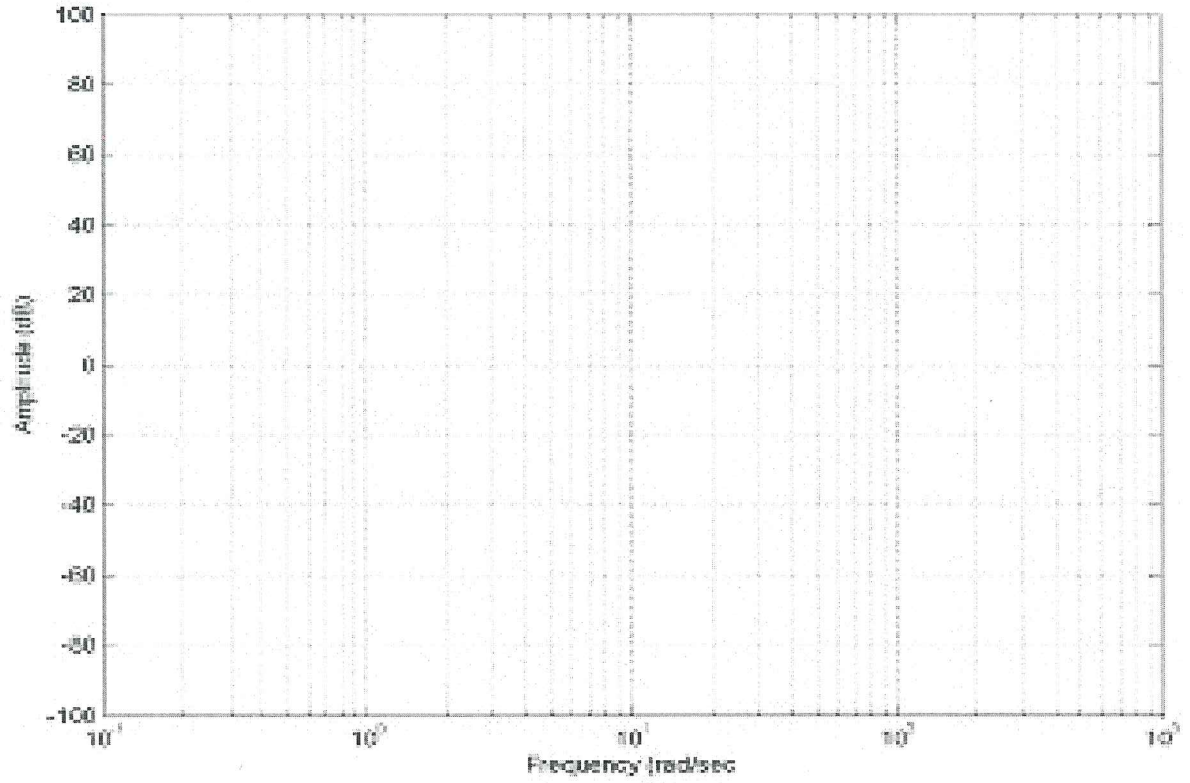
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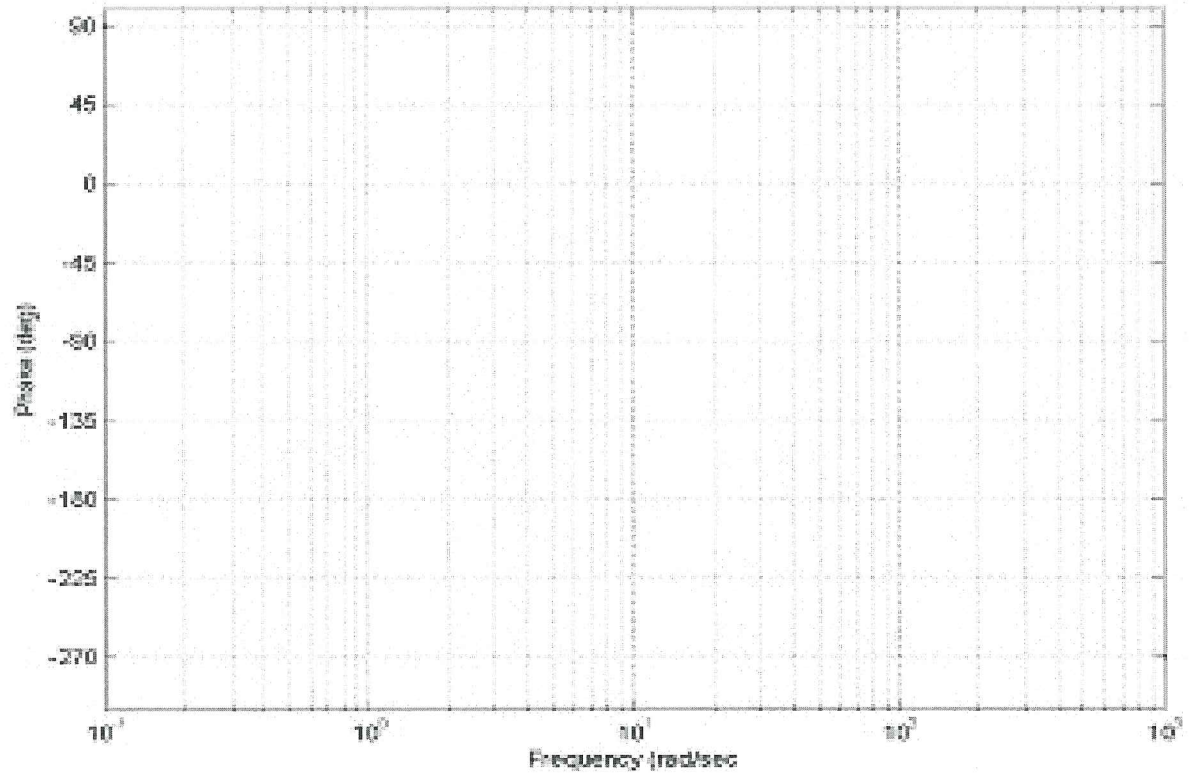
Frequency (rad/sec)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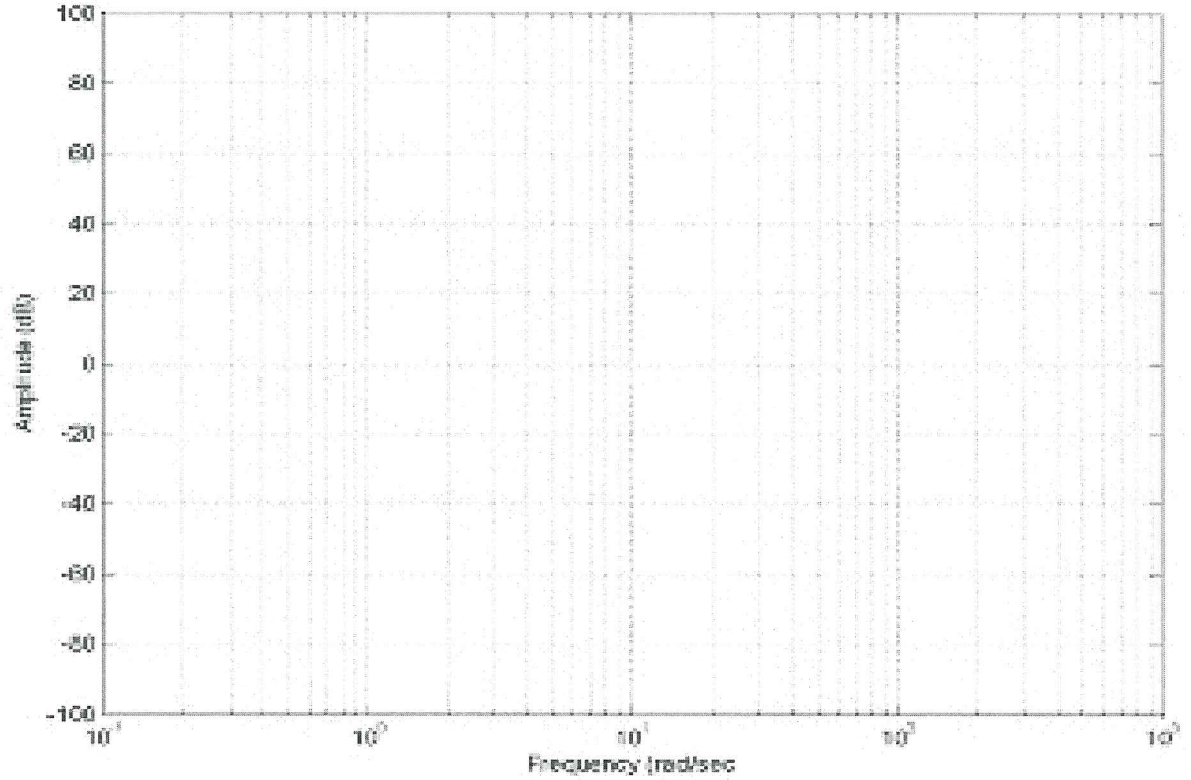
Frequency (rad/sec)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Amplitude (dB)
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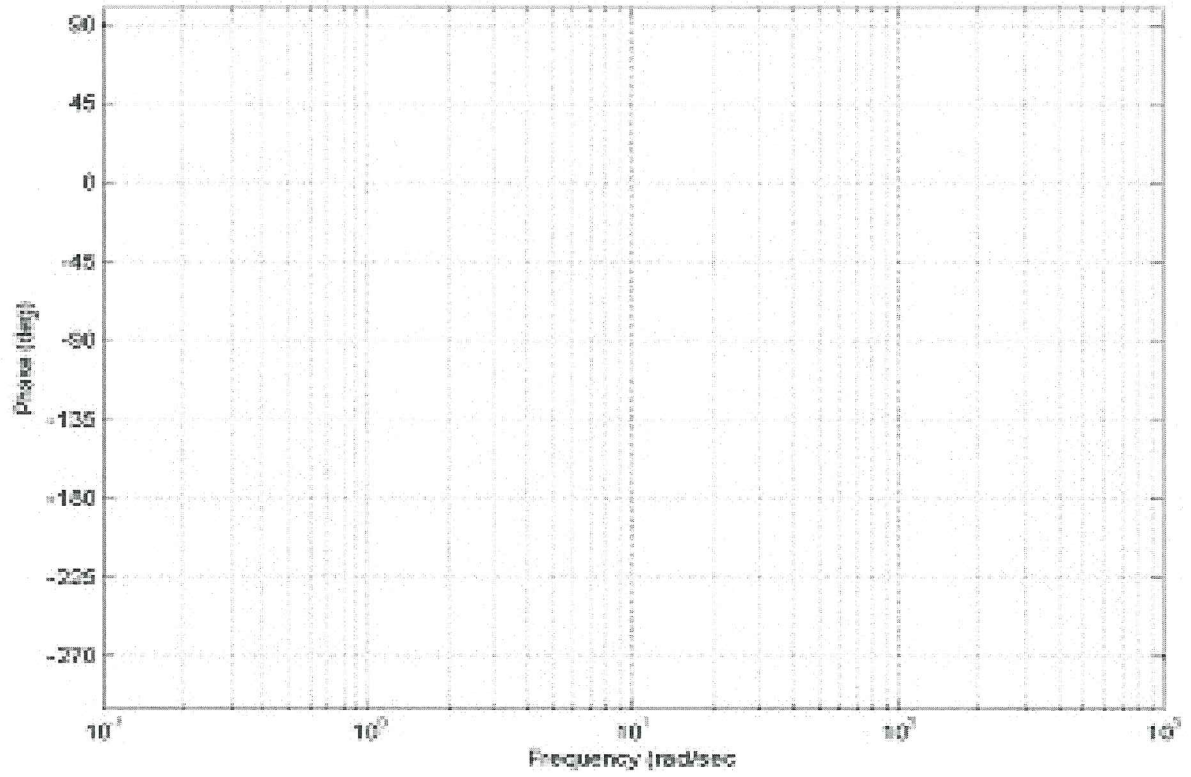
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Phase (deg)
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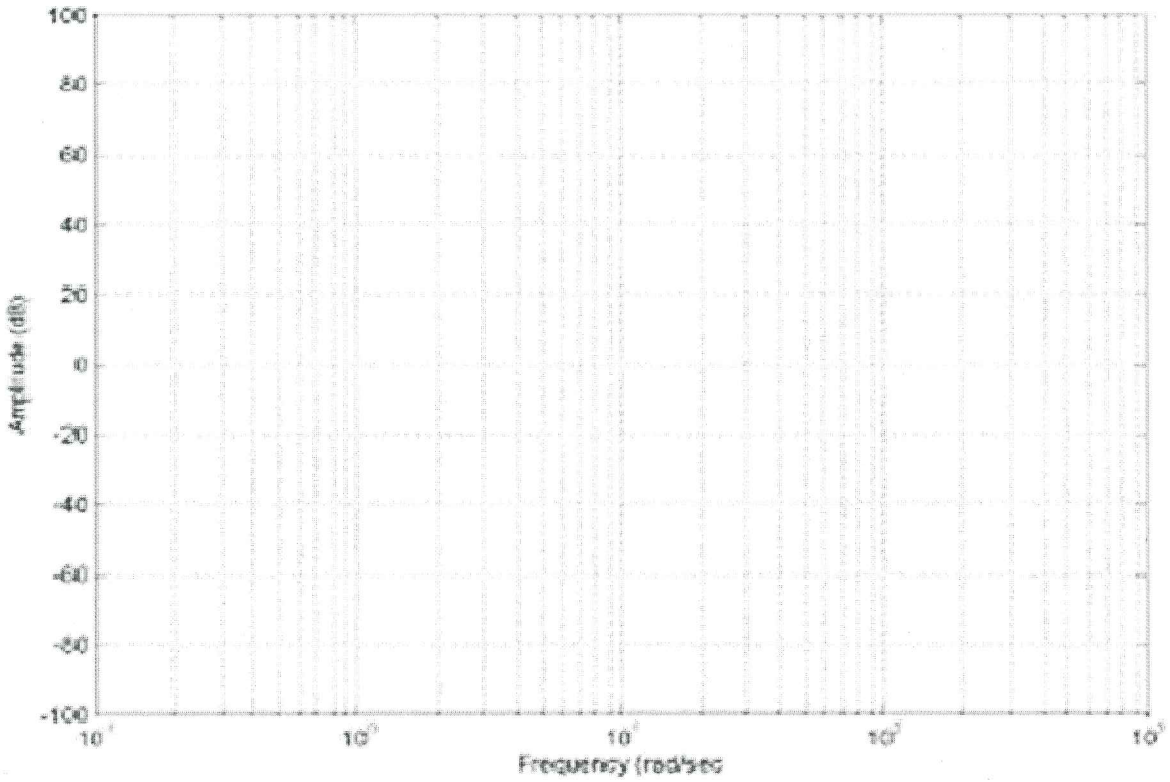
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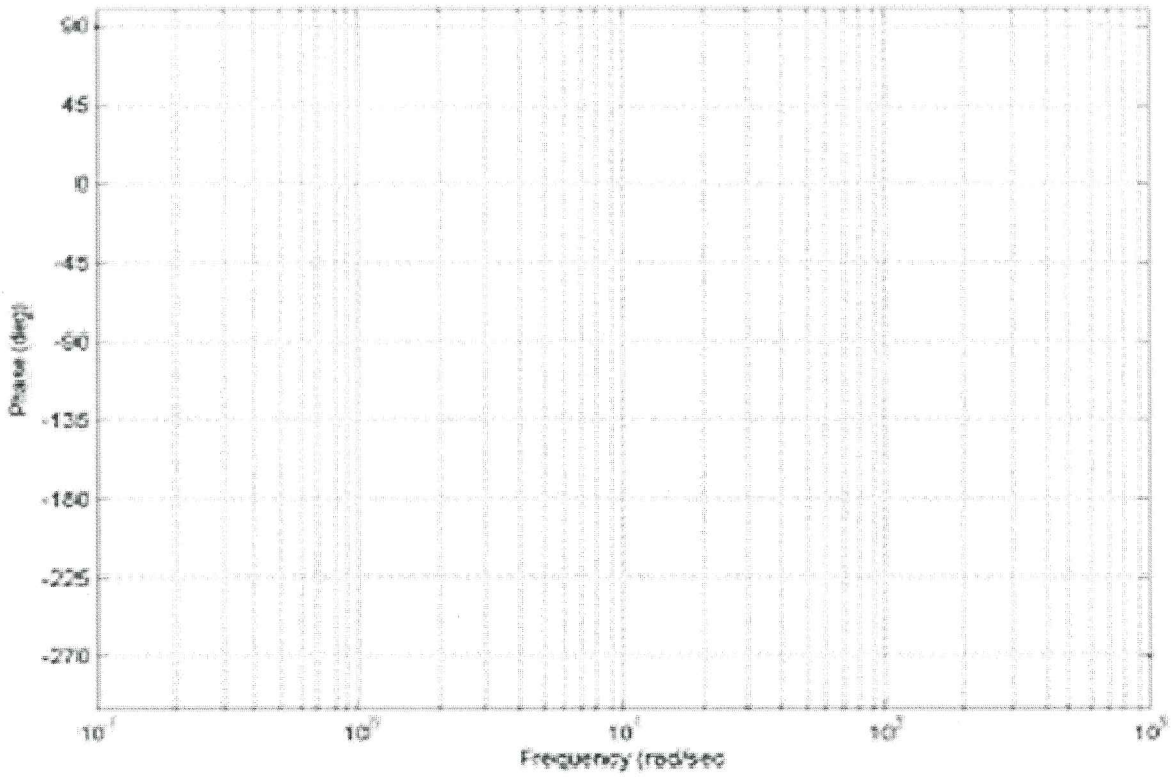
Amplitude (dB)
Frequency (rad/sec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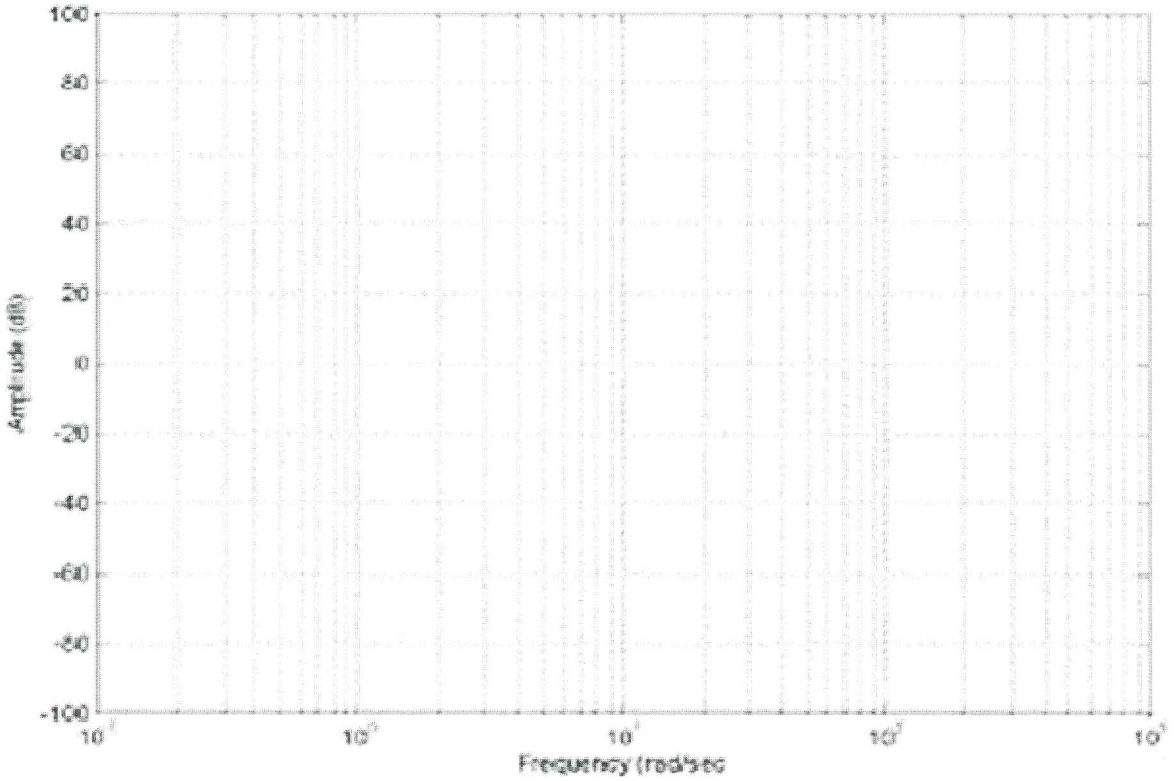
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Amplitude (dB)
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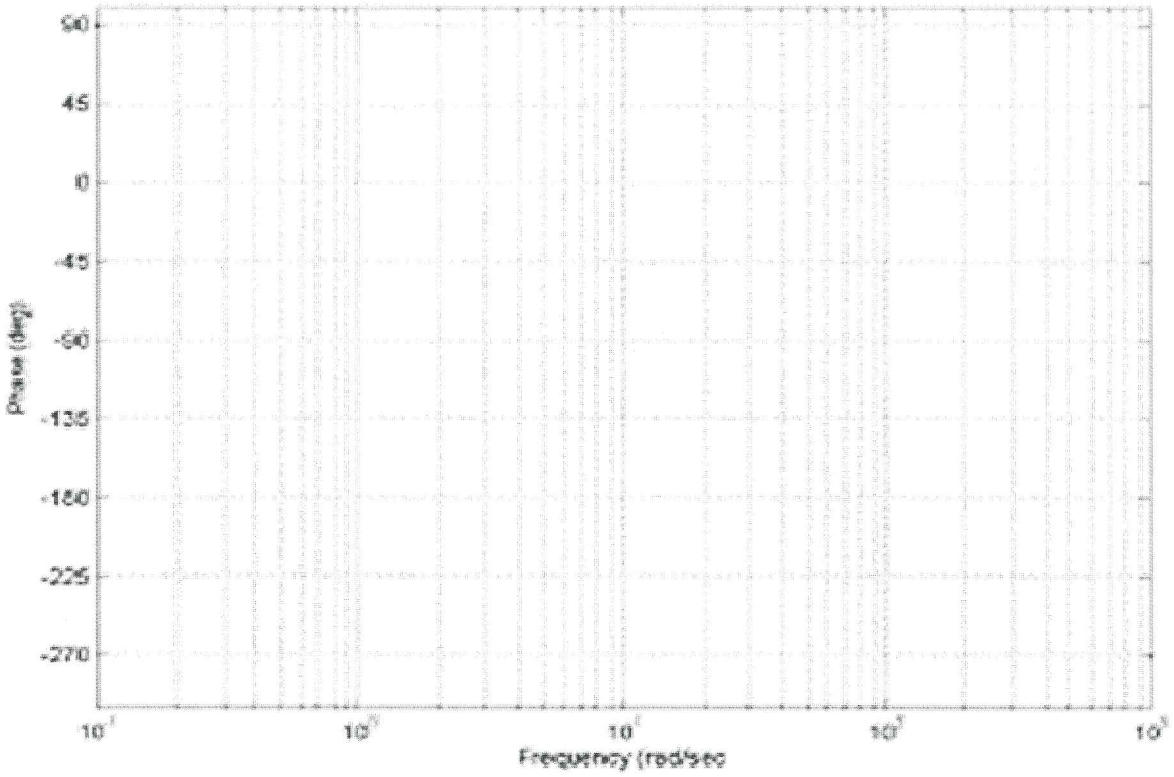
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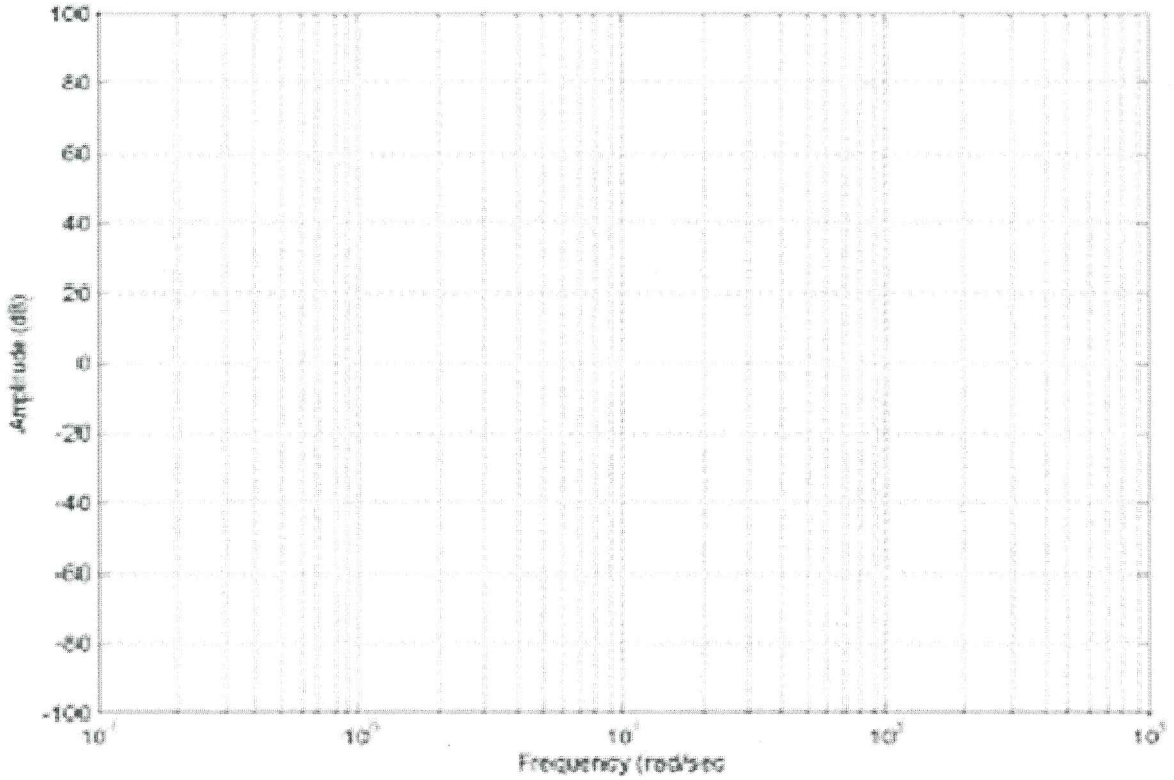
Phase (deg)
Frequency (rad/sec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Amplitude (dB)
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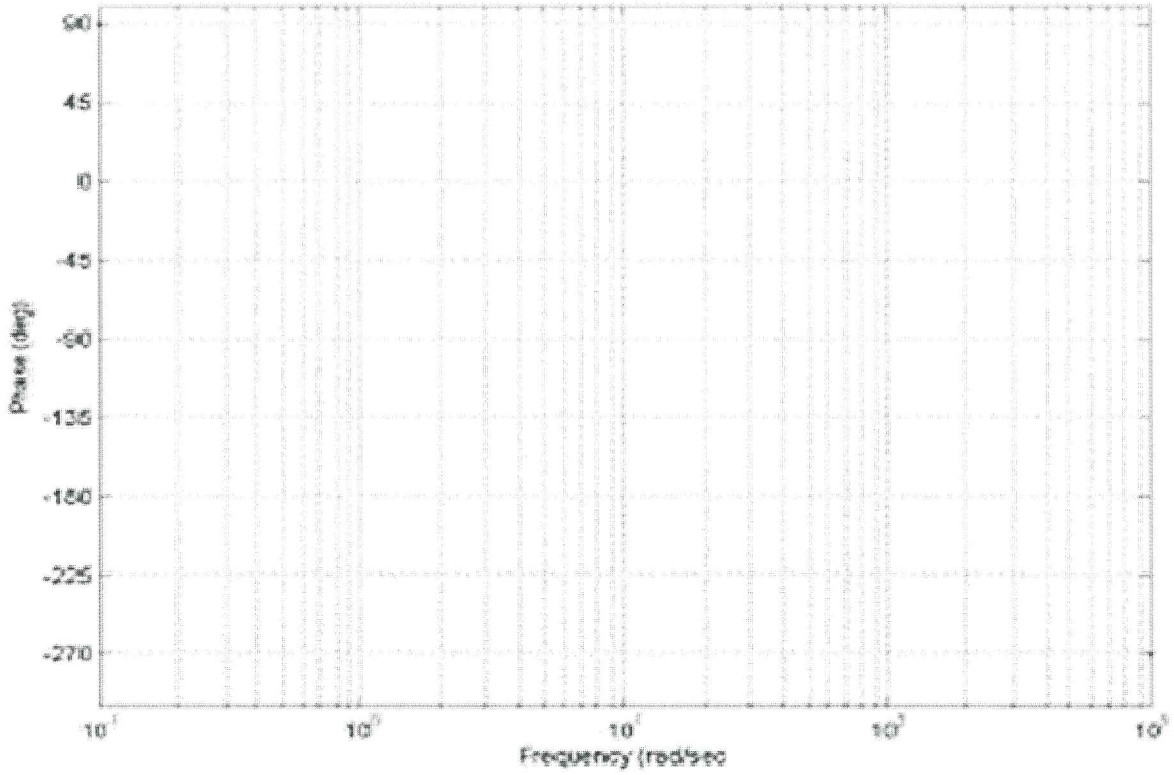
10^3
Frequency (rad/sec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Phase (deg)
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Frequency (rad/sec