UNIVERSITETET I AGDER

Emnekode : MA-154
Kandidatnr. : 127
Dato : 11.12.2012
Ark nr. : 1 av 11

(A)

Oppgave 1

a) $\lim_{x\to 0} \frac{x(\cos x - 1)}{\sin x - x} = \frac{0(\cos 0 - 1)}{\sin 0 - 0} = \frac{0}{0}$

Bruker L'Hopital.

$\lim_{x\to 0} \frac{(\cos x - 1) + x(-\sin x)}{\cos x - 1} = \frac{(\cos x - 1) - x(\sin x)}{\cos x - 1} = \frac{0}{0}$

L'Hopital 2. gang

$\lim_{x\to 0} \frac{-\sin x - (\sin x + x\cos x)}{-\sin x} = \frac{-2\sin x - x\cos x}{-\sin x} = \frac{0}{0}$

L'Hopital 3. gang

$\lim_{x\to 0} \frac{-2\cos x - (\cos x + x(-\sin x)}{-\cos x} = \frac{-3\cos x + x\sin x}{-\cos x}$

$= \frac{-3\cos 0 + 0\sin 0}{-\cos 0} = \frac{-3}{-1} = \underline{\underline{3}}$

b) $y = \frac{1 + \ln x}{x}$

$y' = \frac{\frac{1}{x}\cdot x - (1 + \ln x)\cdot 1}{x^2} = \frac{1 - 1 - \ln x}{x^2} = \underline{\underline{-\frac{\ln x}{x^2}}}$

c) $y = (\sin x)^x$

$\ln y = \ln(\sin x)^x \iff \ln y = x \ln(\sin x)$

$\frac{1}{y} y' = \ln(\sin x) + x \cdot \frac{1}{\sin x} \cdot \cos x$

$\frac{1}{y} y' = \ln(\sin x) + \frac{x\cos x}{\sin x}$

$y' = \left(\ln(\sin x) + \frac{x\cos x}{\sin x}\right) \cdot y \Rightarrow \underline{\underline{\left(\ln(\sin x) + \frac{x\cos x}{\sin x}\right)\cdot(\sin x)^x}}$

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Oppgave 1

I $\quad y^2 = x^2 + \sin(xy) \quad \Big| \frac{d}{dx}$

$$\Rightarrow 2y\frac{dy}{dx} = 2x + \cos(xy)\cdot\left(y + x\frac{dy}{dx}\right)$$

$$2y\frac{dy}{dx} = 2x + y\cos(xy) + x\frac{dy}{dx}\cos(xy)$$

$$2y\frac{dy}{dx} - x\frac{dy}{dx}\cos(xy) = 2x + y\cos(xy)$$

$$\frac{dy}{dx}\left(2y - x\cos(xy)\right) = 2x + y\cos(xy)$$

$$\frac{dy}{dx} = \frac{2x + y\cos(xy)}{2y - x\cos(xy)}$$

II Stigningstallet m i punktet (1, 0)

$$y' = \frac{2\cdot 1 + 0\cos(1\cdot 0)}{2\cdot 0 - 1\cos(1\cdot 0)} = -\frac{2}{1} = -2$$

III Likning til tangent:

$$(y - y_0) = m(x - x_0)$$

$$\Rightarrow (y - 0) = -2(x - 1)$$

$$y = -2x + 2$$

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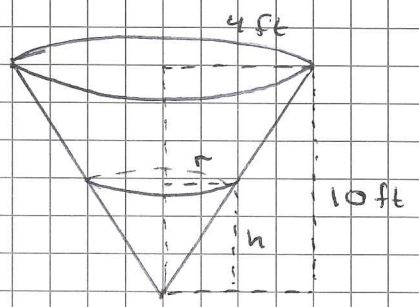
Oppgave 1

e Oplysninger:



$$\frac{dV}{dt} = -5\ ft^3/min$$

$$H = 10\ ft$$

$$R = 4\ ft$$

$$V = \frac{1}{3} r^2 \pi h$$

I Finner forholdet:

$$R = 4,\quad H = 10 \Rightarrow \frac{R}{H} = \frac{r}{h} \Rightarrow \frac{4}{10} = \frac{r}{h} \Rightarrow \underline{\underline{r = \frac{2}{5} h}}$$

II Skal finn ut hvor raskt vannivået synker når vannhøyden er 6 ft, $\frac{dh}{dt}$

$$V = \frac{1}{3} r^2 \pi h \Rightarrow V = \frac{1}{3}\left(\frac{2}{5} h\right)^2 \pi h \Rightarrow V = \frac{4}{75} \pi h^3$$

Deriverer:

$$\frac{dV}{dt} = \frac{4}{75} \pi \cdot 3h^2 \frac{dh}{dt} \Rightarrow -5 = \frac{4}{75} \pi \cdot 3 h^2 \frac{dh}{dt}$$

$$\Rightarrow \frac{dh}{dt} = -\frac{5}{\frac{4}{25} \pi \cdot 6^2} = -\frac{5}{\frac{144\pi}{25}} = -\frac{125}{144\pi} = \underline{\underline{-0,276\ ft/min}}$$

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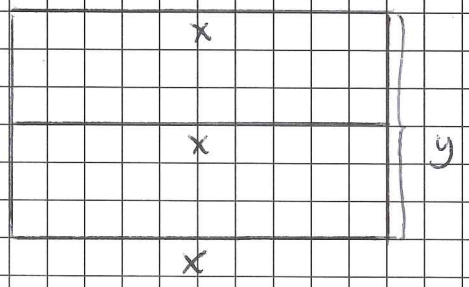
Oppgave 1

f 1. Skal finne hvor mye gjerde han trenger rundt hele området.

2. Skal finn hvor mye gjerde han bruker totalt



Oplysninger

$A = 216\ m^2$

$A = x \cdot y$

$\Rightarrow y = \frac{A}{x}$

$O = 3x + 2y$

$O = 3x + 2 \cdot \left(\frac{A}{x}\right)$

$O = 3x + \frac{432}{x} \quad | \frac{d}{dx}$

$\frac{dO}{dx} = 3 + \left(-\frac{432}{x^2}\right)$

$O' = 3 - \frac{432}{x^2} = 0$, setter det lik null for å finne max verdi

$-\frac{432}{x^2} = -3 \Rightarrow x^2 = \frac{432}{3} \Rightarrow x = \pm\sqrt{\frac{432}{3}}$

$x = 12 \vee x = -12$

$\Rightarrow y = \frac{216}{12} = 18 \Longrightarrow y = 18$

1. Det ytre gjerdet må være: $O = 2x + 2y = 60\ m$
2. Det totale gjerde må være: $O = 3x + 2y = 72\ m$

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Oppgave 2

a) $\int \frac{x^3}{x^2-2x+1}\,dx$

Deler

$x^3 : (x^2-2x+1) = x - 2 + \frac{5x-2}{x^2-2x+1}$

$x^3 - 2x^2 + x$

$-2x^2 + x$

$-2x^2 + 4x - 2$

$5x - 2$

$\Rightarrow \int \frac{x^3}{x^2-2x+1}\,dx = \int x-2\,dx + \int \frac{5x-2}{(x-1)^2}\,dx$

$\Rightarrow \int \frac{5x-2}{(x-1)^2}\,dx \rightarrow$ MÅ LØSE DENNE FØRST → DELBRØKSOPPSPALTING

$\frac{5x-2}{(x-1)^2} = \frac{A}{x-1} + \frac{B}{(x-1)^2} \quad | \cdot (x-1)^2$

$\Rightarrow 5x - 2 = A(x-1) + B$

$= Ax - A + B$

$A = 5 \quad \vee \quad -A + B = -2$

$-5 + B = -2$

$B = 3$

$\Rightarrow \int (x-2)\,dx + \int \frac{5}{(x-1)}\,dx + \int \frac{3}{(x-1)^2}\,dx$

$= \left[\frac{1}{2}x^2 - 2x + 5\ln|x-1| + 3(x-1)^{-1}\right] + C$

$= \left[\frac{1}{2}x^2 - 2x + 5\ln|x-1| - \frac{3}{x-1}\right] + C$

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Oppgave 2

b $$\int_0^2\int_0^1 (x+y)\,dy\,dx = \int_0^2\left(\int_0^1 (x+y)\,dy\right)dx$$

$$= \int_0^2\left[xy+\tfrac{1}{2}y^2\right]_0^1 dx = \int_0^2\left(x\cdot 1+\tfrac{1}{2}1^2\right)dx = \int_0^2\left(x+\tfrac{1}{2}\right)dx$$

$$= \left[\tfrac{1}{2}x^2+\tfrac{1}{2}x\right]_0^2 = \left(\tfrac{1}{2}\cdot 2^2+\tfrac{1}{2}\cdot 2-\left(\tfrac{1}{2}\cdot 0^2+\tfrac{1}{2}\cdot 0\right)\right)$$

$$= 2+1 = \underline{3}$$

c Finn 2. ordens partiellderivert

$$f(x,y) = 2x^3+6xy^2-3y^3-150x$$

$$f_x = 6x^2+6y^2-150$$

$$f_{xx} = \underline{12x}$$

$$f_{xy} = \underline{12y}$$

$$f_y = 12xy-9y^2$$

$$f_{yy} = \underline{12x-18y}$$

$$f_{yx} = \underline{12y}$$

$\Rightarrow$ 2. ordens partiellderiverte er $\underline{f_{xx}=12x,\ f_{yy}=12x-18y,\ f_{xy}=f_{yx}=12y}$

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Oppgave 3

a) $\sum_{n=0}^{\infty} \frac{x^n}{\sqrt{n^2+3}}$ → Velger forholdstest , $a_n = \frac{x^n}{\sqrt{n^2+3}}$

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \frac{\frac{x^{n+1}}{\sqrt{(n+1)^2+3}}}{\frac{x^n}{\sqrt{n^2+3}}} = \lim_{n\to\infty}\frac{x^n \cdot x \cdot \sqrt{n^2+3}}{x^n\sqrt{(n+1)^2+3}} =$$

$$= \lim_{n\to\infty}\left(|x|\frac{\sqrt{n^2+3}}{\sqrt{(n+1)^2+3}}\right) = \lim_{n\to\infty}\left(|x|\cdot\underbrace{\sqrt{\frac{n^2+3}{(n+1)^2+3}}}_{\text{går mot 1}}\right)$$

For holdstest sier:

$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| < 1$, siden $\sqrt{\frac{n^2+3}{(n+1)^3+3}} \approx 1$, får vi ulikheten;

$|x| < 1$, Løser så denne:

$x < 1$ V $x > -1$

"konvergens"

$-1 < x < 1$

Sjekker endepunkt

$x = -1$ $\sum_{n=0}^{\infty} \frac{(-1)^n}{\sqrt{n^2+3}} = \sum_{n=0}^{\infty}(-1)^n \frac{1}{\sqrt{n^2+3}}$

Alternerende rekketest

1. $a_n > 0$ → Ok
2. $a_n \geq a_{n+1}$ → Ok
3. $\lim_{n\to\infty} a_n = 0$ → Ok

} Betinget konvergens

Forts. neste ark.

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Oppgave 3

a) Må undersøke om $\sum_{n=0}^{\infty} a_n$ konvergerer

$$\sum_{n=0}^{\infty} \frac{1}{\sqrt{n^2+3}}$$

$x = 1$: $\sum_{n=0}^{\infty} \frac{1}{\sqrt{n^2+3}}$ $\Rightarrow$ Integraltest $\Rightarrow \int_0^R \frac{1}{\sqrt{n^2+3}}\,dn = \int_0^R (n^2+3)^{-\frac{1}{2}}\,dn$, $R = \infty$

$$= \left[4n(n^2+3)^{\frac{1}{2}}\right]_0^R = 4R\sqrt{R^2+3} = \infty$$

$\downarrow$

Divergens

Da har vi:

(I) konvergensintervall: $-1 \leq x < 1$

(II) Absolutt konvergens: $-1 < x < 1$

(III) Betinget konvergens: $x = -1$

(IV) Rekkas senter: 0

(V) Rekkas radius: 1

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Oppgave 3

b Taylors formel:

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f^{n+1}(a)}{(n+1)!}(x-a)^{(n+1)}$$

Skal finne fire ledd av

$f(x) = 5\cos \pi x$, $x = 0$

Tabell:

$f(x) = 5\cos(\pi x) \rightarrow f(0) = \underline{5}$

$f'(x) = 5\sin(\pi x)\cdot \pi \rightarrow f'(0) = 0$

$f''(x) = -5\pi^2\cos(\pi x) \rightarrow f''(0) = \underline{-5\pi^2}$

$f'''(x) = -5\pi^3\sin(\pi x) \rightarrow f'''(0) = 0$

$f^4(x) = 5\pi^4\cos(\pi x) \rightarrow f^4(0) = \underline{5\pi^4}$

$f^5(x) = 5\pi^5\sin(\pi x) \rightarrow f^5(0) = 0$

$f^6(x) = -5\pi^6\cos(\pi x) \rightarrow f^6(0) = \underline{-5\pi^6}$

$$\Rightarrow f(x) = 5 + 0 + \frac{(-5\pi^2)}{2!}(x-0)^2 + 0 + \frac{5\pi^4}{4!}(x-0)^4 + \frac{(-5\pi^6)}{6!}(x-0)^6$$

$$= 5 - \frac{5}{2}\pi^2x^2 + \frac{5}{24}\pi^4x^4 - \frac{5}{720}\pi^6x^6$$

$$= \underline{5 - \frac{5}{2}(\pi x)^2 + \frac{5}{24}(\pi x)^4 - \frac{5}{720}(\pi x)^6}$$

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Oppgave 4

a $z = -1 + i$

I $Re = -1$

$Im = 1$

II Modulus

$r = |z| = \sqrt{(-1)^2 + 1^2} = \underline{\underline{\sqrt{2}}}$

Argumentet

$\theta = \arg(z)$

$\tan\theta = \frac{1}{-1} \Leftrightarrow \tan\theta = -1$

$\theta = \tan^{-1}(-1)$

$\underline{\underline{\theta = -0{,}785}} \Leftrightarrow \underline{\underline{\theta = -\frac{\pi}{4}}}$

b Forenkle

$$z = \frac{3+2i}{2+2i} = \frac{(3+2i)(2-2i)}{(2+2i)(2-2i)} = \frac{6-6i+4i-4i^2}{4-4i^2+4i-4i}$$

$$= \frac{6-2i+4}{4} = \frac{10-2i}{4} = \frac{5-i}{2} = \underline{\underline{\frac{5}{2} - \frac{1}{2}i}}$$

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Oppgave 4

c Linearisering

$L(x,y) = f(x_0,y_0) + f_x(x_0,y_0)(x-x_0) + f_y(x_0,y_0)(y-y_0)$

$f(x,y) = (x+y+2)^2$, $(1,2)$

Noen mellomregninger:

$f(1,2) = (1+2+2)^2 = \underline{25}$

$f_{x(x,y)} = \cancel{X} + \cancel{X} = 2(x+y+2)\cdot 1 = 2(x+y+2)$

$f_{x(1,2)} = \underline{\cancel{X}} = 2(1+2+2) = \underline{10}$

$f_{y(x,y)} = 2(x+y+2)\cdot 1 = 2(x+y+2)$

$f_{y(1,2)} = 2(1+2+2) = \underline{10}$

$L(1,2) = 25 + 10(x-1) + 10(y-2)$

$= 25 + 10x - 10 + 10y - 20$

$= 10x + 10y - 5$

$= \underline{5(2x+2y-1)}$

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