Agder University College
Department of mathematics and science
Nordic Graduate School in Mathematics Education
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20070429

Self-evaluation of the Nordic Research Schools in the Humanities and Social Sciences 2004-2008

Nordic Graduate School in Mathematics Education

1 General information on the Nordic Graduate School in Mathematics Education (NoGSME)

Director of NoGSME is Professor Barbro Grevholm, Agder University College, Kristiansand, Norway.

The Board of NoGSME
consists of the director and the following members:
Christer Bergsten, University of Linköping, Sweden
Ole Björkqvist, Abo Akademi University, Vasa, Finland
Trygve Breiteig, Agder University College, Kristiansand, Norway
Mogens Niss, Roskilde University Center, Denmark
Guðný Helga Gunnarsdóttir, Iceland University of Education, Reykjavik, Iceland –and finally
Madis Lepik, University of Tallinn, Estonia as observer participant for the Baltic countries

Participating institutions
Nordic and Baltic partners in the co-operation

Denmark (universities and contact persons)
Roskilde University Center, Mogens Niss
Danish University of Education, Jeppe Skott
Aalborg University, Ole Skovsmose
University of Southern Denmark, Claus Michelsen
University of Copenhagen, Carl Winslöw

Finland
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University of Helsinki, Erkki Pehkonen
University of Turku, Kaarina Merenluoto
University of Joensu, Tuomas Sorvali
University of Jyväskylä, Pekka Kupari
University of Lapland, Raimo Kaasila
University of Oulu, Tapio Keranto
University of Tampere, Harry Silfverberg

Iceland
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Norway
Agder University College, Barbro Grevholm and Trygve Breiteig

University of Oslo, Gunnar Gjone
Oslo University College, Bodil Kleve
Narvik University College, Ragnhild Johanne Rensåå
Norwegian University of Science and Technology, Ingvill Stedøy Johansen
Sør-Trøndelag University College, Frode Rønning, Geir Botten
Stavanger University, Elin Reikerås and Reidar Mosvold
Bergen University, Ole Einar Torkildsen,
Bergen University College, Marit Johnsen Høines
University of Tromsø, Anne Fyhn
Telemark University College, Gard Brekke and Åse Streitlien
Volda University College, Ole Einar Torkildsen
University College of Bodø, Tone Bulien

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University of Gothenburg, Roger Säljö
Kristianstad University College, Ingemar Holgersson
University of Umeå, Johan Lithner
University of Linköping, Christer Bergsten
University of Uppsala, Anders Öberg
University of Stockholm, Claes Löfwall
Royal University of Technology in Stockholm, Ambjörn Naeve
University College of Dalarna, Eva Taflin
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Mette Andresen	Danish University of Education
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Markku Hannula	University of Tallinn
Latvia	
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Edvins Gingulis	Liepaja Academy of Pedagogy
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Lithuania	
Arkadijus Kiseliovas	Siauliai University
Ricardas Kudzma	Vilnius University

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Evija Slokenberga	University of Riga
Ilva Magazeina	University of Riga

Study plans and literature

Courses initiated by NoGSME and given in collaboration with one of the participating institutions:

Didactics of mathematics – The French way, Copenhagen University, Carl Winslöw, autumn 2004-spring 2005. Course description is presented in a separate file 4f.pdf.

Views of knowing and learning: Constructivism, socio-cultural theory and the relationships between them, with special emphasis on the writings of Paul Cobb, Copenhagen Pedagogical University, Professor Jeppe Skott, autumn 2005. Course-description and literature are presented in appendix 3.

Mathematics and gender, National Centre for Mathematics Learning, Norwegian University of Science and Technology, Ingvill Stedöy-Johansen, spring 2006. Course-description and literature are presented in appendix 4.

Justification in mathematics and science education research with special emphasis on the role of theory in justification. Danish Graduate School in mathematics and science education, Copenhagen, Mogens Niss, autumn 2007.

NoGSME is collaborating with AUC and is thus able to offer the regular ph d courses at AUC:

MA-601 Theory of Science from a perspective of Mathematics Education

MA-602 Theories in the learning and teaching of Mathematics

MA-603 Meta-perspectives on mathematics and the learning of mathematics in a technologically powerful environment

MA-604 Problem-solving

MA-606 History of mathematics with emphasis on modern Mathematics

MA-607 Research Methodology in Mathematics Education

Here follow the study plans of the AUC-courses (lists of literature are in Appendix 2):

Code: **MA-601**
Course title: **Theory of Science from a perspective of Mathematice Education**
Credits: 5
ECTS-credits: 5
Language: English
Course description:
Main issues will be:

1 What is science? What is science generally? What is mathematics as a science? What is mathematics education as a science? The historical development will be illuminated.
2 Science, humanity and social science. Mathematics education as a multi-disciplinary or inter-disciplinary area.
3 Theories in mathematics education. Different theories, models and hypotheses in mathematics education.
4 Truth, understanding and method. What is the meaning of truth in mathematics education? How are the concepts understanding and meaning treated?
5 Science and values Values and valuations in mathematics education. Ethical questions.
6 Mathematics education and society: Political questions and research in mathematics. Justification of research.
Aim and objective:
The aims and objectives of the course is to

- offer the participants opportunities to develop knowledge about scientific research and development from a historical, social and philosophical perspective with special respect to mathematics education.
- offer opportunities to encounter central issues about scientific methods and ideals in mathematics education.
- increase the ability of the participants to relate their own thesis work to the theory of science and to historical and social perspectives on mathematics education.

Teaching: Lectures, seminars
Duration/semester(s): One semester
Assessment: Research project or essay
Credit overlap: See Student's Handbook

Code: **MA-602**
Course title: **Theories in the learning and teaching of Mathematics**
Credits: 10
ECTS-credits: 10
Language: English
Course description: The course will start off from questions about mathematics itself – for example, the nature of mathematics and of mathematical activity and thinking – and move into questions of what it means to learn and to teach mathematics.
This will take us into learning theory or theories. We will consider very briefly theories such as associationism (behaviourism) and rationalism (naturism) but move rapidly into constructivism and socio-cultural theory as being the two areas of theory which have influenced mathematics education most obviously in the last two decades. This will take us into the work of Piaget and Vygotsky and their associates and followers.
We will look at some examples of learner's activity and thinking and try to use theories as lenses to explain and situate learning as we see it, and will go on to talk about analysing learning through classroom observation and interview, tests and experiments.
Consideration of the classroom will open up questions about teaching and how teaching is related to learning. How are approaches to teaching linked to teachers' perspectives on mathematics and on the learning of mathematics? In what ways can theories of learning inform teaching? What is involved in the process of teaching and how might this be seen to relate to the education of teachers of mathematics.

What current theories provide insights into teaching processes and ways in which teaching can develop? In what ways can theories of teaching inform the practice of teaching so that students' learning mathematics can have a richer, more fruitful and successful learning experience?
What do we regard as successful learning and teaching; how is this reflected in mathematical performance; and what are the socio-cultural, economic and political issues behind such judgments? Such considerations will take us into areas such as language and discourse, equity and systemic issues in the organization of teaching and learning.
Aim and objective: To introduce, discuss and debate theories of mathematics learning and teaching, with relation to perspectives of knowledge in mathematics and mathematics education, and as fundamentally related to associated practices. This will include critical discussion of what we mean by *theory* and how theories in the public domain are related to practitioners' personal theories and to practices in mathematics learning and teaching at all levels.
Teaching: Lectures, seminars
Duration/semester(s): One semester
Assessment: Essay
Credit overlap: See Student's Handbook

Code: **MA-603**
Course title: **Meta-perspectives on mathematics and the learning of mathematics in a technologically powerful environment**
Credits: 10
ECTS-credits: 10
Language: English
Course description: Main research question: What does it mean, or can it mean, to study mathematics and the learning of mathematics in a powerful technological environment? The course opens an opportunity to function in a user-friendly technological environment for collaboratively constructing knowledge that may contribute to answering the main question of the course.
There are three sessions where the participants meet at AUC, two sessions of two or three days in the start and near the end, and one main session for ten days midway through the semester. During that session there will be several guests through videoconferences. The participants will have read some of their writings and become somewhat acquainted with their work. They will have prepared themselves collaboratively for questioning and discussing with the guests, themes and aspects that are of particular importance or interest to them, as well as the inputs the guests choose to make. After each videoconference there are sessions for reflecting, conceptualising and beginning of writing.
Content of the course
As the frame for the course is very wide it is necessary to specify and narrow the content which is considered. The first time the course is running this is done in the following way:
- The mathematical content in focus is patterns and algebra.
- The important mathematics learning phenomena in focus are concept constructing and collaborative efforts in understanding.

• The technologically interesting questions are about the ease of attainability of material which can throw light on historical development and applicability of particular mathematics as well as open up for exploring and connecting concepts and ideas.
Aim and objective: The aim of the course is to give the participants opportunity to
• study carefully what it can mean to study mathematics and the learning of mathematics in a technologically powerful environment.
• study similarities and differences in both visions and methodology of mathematics learning, as can be seen in, on one hand, general writings within the field of mathematics education, and on the other hand in writings of promoters for using technology
• offer the participants opportunity to work collaboratively in constructing knowledge and have an opportunity to meet with experts in the area through carefully planned videoconferencing
Teaching: Lectures, discussions both in presence and at distance through web and video-conferencing. Collaboration through projects within small groups as well as by all the participants.
Duration/semester(s): One semester
Assessment: Each individual will be assessed twice through individual writings connected to obligatory and personally chosen reading material. Groups will be assessed twice for collaborative projects, first time in preparing the main session and secondly in the resulting work from the main session.
Credit overlap: See Student's Handbook

Code: **MA-604**
Course title: **Problem Solving**
Credits: 10
ECTS-credits: 10
Language: English
Course description: The course will discuss problem solving in mathematics, both used individually and collaboratively in small groups, with focus on:
- the problem-solving process, involving cognitive, metacognitive, socio-cultural and affective processes,
- collaborative problem solving in small groups
- problem solving as a tool for learning and teaching at different levels in school,
- the development of problem-solving research
- methods for research in collaborative problem solving in small groups/class,
- research literature.
Aim and objective:
The aim of this course is, to develop a deeper understanding of problem solving and collaborative small- group work in mathematics such that it can be used professionally in teaching mathematics and in mathematics education research.
Teaching: Lectures, seminars
Duration/semester(s): One semester
Assessment: Oral exam, research project or essay
Credit overlap: See Student's Handbook

Code: **MA-606**
Course title: **History of mathematics with emphasis on modern Mathematics**
Credits: 10
ECTS-credits: 10
Language: English
Course description: Main issues in the course will be:
Mathematics in the 19th century as presupposition for the revolution in mathematics around 1900, as the breakthrough in algebra, number concept, non-Euclidean geometry and set theory.
Mathematics around 1900 focusing on the cognitive and communicative revolution.
Mathematics around 1940 with emphasis on political misuse in some societies, military use, the start of structure mathematics and of informatics.
Mathematics around 2000: Specializing, standardizing, change in concept of mathematical proof, mathematics education in historical perspective
Aim and objective: The aim and objective of this course is in coincidence with to that of the course MA-403.
The aim of this course for the doctoral students is, in the first place, to approach the historical development of mathematics up to our time, study some main strands and pre-suppositions for development, and to focus on its relations to society and to education.
Teaching: Lectures, seminars, individual reading
Duration/semester(s): One semester
Assessment: Written exam with graded evaluation
Credit overlap: See Student's Handbook

MA-607 Research Methodology in Mathematics Education

15 ECTS credits - 1 semester - Spring

Study programmes
Mathematics Education, Ph.D. Programme

Recommended previous knowledge
Participants should have a master in mathematics education or mathematics and some knowledge on, and background in, didactics of mathematics (or equal qualifications).

Language of instruction
English.

Objective
The course will address the nature of research in mathematics education, starting from epistemological perspectives. It will introduce and explore a range of research paradigms with associated qualitative and quantitative approaches and methods. Key approaches and methods will be discussed in detail alongside examples of their use from real research studies. There will be a practical dimension in which approaches and techniques in data collection and analysis, including use of relevant software, will be demonstrated and discussed. Research ethics, validation of results and presentation of findings will be important considerations. The course will be centrally related to the research areas of the

participants and participants will be required to use their own research focuses as a basis for discussing research approaches and methods. Participants will be expected to develop expertise in reading research that uses methods beyond those used in their own research. The course has a wide reading list and participants will be encouraged to read centrally indicated key texts, to select for themselves from the wider list and to find their own articles by using established search techniques.

Aim
To develop an understanding of research in mathematics education from epistemological, theoretical and practical perspectives.
To experience a wide range of research approaches and methods and understand their use and application.
To develop a methodological approach to one's own research and an ability to read and understand reports of research more widely.
To develop the capacity to be reflective, analytic and critical researchers.

Type of course
Theoretical.

Teaching methods
Instruction is comprised of lectures, seminars and workshops. There will be 5 contact weeks.

Examination
Essay. Pass/Fail.

Credit reduction
MA-605 with 10 credits

The lists of literature to the courses MA-601 to MA-607 can be found in Appendix 2.

2 Report on the activities and results 2004-2006/2007 (max 10 pages in total)

Activity description (max 4 pages)
NoGSME is offering activities for both doctoral students and supervisors and lately also prospective supervisors, as there are many recent doctors in mathematics education who will become the future supervisors.
The activities for students are doctoral courses, summer schools, and research workshops. They can also apply for mobility stipends and economic support for travels to courses.
For the supervisors we offer seminars aiming for professional development of supervisors and research workshops. NoGSME invites supervisors to create courses and cooperate in the planning of activites.
Over the year 2004-2007 NoGSME has offered seven seminars for supervisors and four research workshop. There have been a number of courses each semester and a summer school each year. This is all according to the program promised in the application to NordForsk in 2003.

Doctoral Courses in mathematic education
During 2004 NoGSME offered the courses MA-603, MA-605 (spring), MA-601, MA-602 and MA-606 (autumn), which are developed at Agder University College (AUC) and part of the doctoral programme there. In all of the courses some NoGSME students, who are not doctoral students at AUC took part and they all got travel support from NoGSME.
Starting in autumn 2004 and continuing during spring 2005 the course *Didactics of mathematics – The French way* took place at Copenhagen University. Professor Carl Winsløw, who was planning and running the course, also produced documentation of this course, which we have sent to NordForsk in order to give a picture of the work. Two other courses were offered in spring 2005 by the mathematics didactics group at HiA, the course Ma605 *Research design and methods in mathematics education* and MA604 *Problem-solving*. These courses are given on a regular basis and the agreement for such courses is that NoGSME contributes only with a symbolic amount of money to the course. Inevitably, the courses that are initiated in cooperation with NoGSME, developed and given for the first time cost more. In spite of this the board finds it important to support such initiatives as it means that courses can be given for Nordic students, courses which would otherwise not be available at all. It seems as if the departments at different universities in the NoGSME network have realised that there is an opportunity to offer something special thanks to this cooperation.
During autumn 2005 we supported a course on classroom research at Copenhagen Pedagogical University as collaboration with Professor Jeppe Skott. The eminent mathematic didactics researcher Paul Cobb from Tennessee, was central in the course, which dealt with his writings and where he made a contribution himself. This course attracted 23 participants and most from Nordic countries other than Denmark. The students reported that the course was highly valuable for them. All students who requested were provided with travel support for the attendance at this course. Three further courses at AUC were also offered, MA601 *Theory of science from a mathematics education perspective*, MA602 *Theories of learning and teaching mathematics* and MA-606 *Mathematics history*. All courses during 2005 attracted Nordic participants from outside the home university except for the history course.

Two courses were offered in spring 2006 by the mathematics didactics group at Agder University College (AUC), the course Ma-607 *Research methodology in mathematics education* and MA-604 *Problem solving*. A new course was given during spring 2006 at the National Center for Mathematics Education (NSMO) in Trondheim, initiated by the board of NoGSME and developed in cooperation with NSMO. The theme of the course was *Mathematics and gender* and one of the teachers was the well known international researcher, Professor Paul Ernest from Exeter University. Course leader was Ingvill Stedøy-Johansen.
During autumn 2006 we supported some students with travel costs to the course on *Perspectives on Identity in Learning and Education Research* given at Ålborg University and led by Paola Valero. International researchers Anna Sfard and Etienne Wenger were teachers in this course. Both courses attracted participants from Nordic countries other than Norway and Denmark, respectively. The students reported that the courses were highly valuable for them. All students who requested were provided with travel support for the attendance at these courses. Three further courses at AUC have also been offered, MA-601 *Theory of science from a perspective of mathematics education*, MA-602 *Theories of learning and teaching mathematics* and MA-606 *History of mathematics with emphasis on modern mathematics*. All courses during 2006 attracted Nordic participants from outside the home university except for the history course.
During spring 2007 the course on *Research methodology in mathematics education* was given again and this time it attracted 16 participants. Most of them come from the Nordic community of doctoral students outside AUC. For autumn 2007 NoGSME has offered the course MA-606, which will now have students from the Nordic community, and a new course on *Justification in mathematics and science education research with special emphasis on the role of theory in justification*. The courses MA-601 and MA-602 cannot be given this autumn semester because too few students want to take the course.

Summer schools in mathematics education
In 2004 NoGSME collaborated with ERME, The European Society for Research in Mathematics Education and was thus able to offer a summer school in Prague to students. The director of NoGSME was invited as observer participant to learn from the ERME experiences of summer schools and three Nordic students took part. These students promised to be part of the organising committee for the next NoGSME summer school, which took part in University of Jyväskylää in 2005 and had about 30 participants. In 2006 the summer school was in Dömmesmoen in Norway, part of the campus of AUC.
The summer school was able to accept all applicants as we were allowed to use the funds unspent from 2005. Twenty Nordic students took part, well distributed from all the countries. The international experts were Kath Hart from United Kingdom, Ole Björkqvist from Finland, Christer Bergsten from Sweden, Trygve Breiteig from Norway. Lectures were also given by Simon Goodchild, Trygve Breiteig and Barbro Grevholm from HiA, Christer Bergsten from Sweden and Ole Björkqvist from Finland. Thus the summer school had an international flavour.
In 2007 the summer school will be in Laugarvatn in Iceland, part of the campus of Iceland University of Education. There will be 39 participating doctoral students and four group leaders, who are recognised international scholars in the field. The working group leaders are Abraham Arcavi, Marianna Bosch, Marcelo Borba and Eva Jablonka. Four board members will also contribute to the programme of the summer school with workshops and

other parts. In order to strengthen the group in mathematics education at Iceland University of Education three teacher educators have been invited to take part. The summer schools were evaluated with written questionnaires and open discussions by students after each of them and the results show that the students highly appreciate this activity and claim that it is most useful for them in their research project. It is stimulating to be able to discuss your own work during a whole week with interested peers and international scholars. The opportunity to get individual supervision from the leaders is also estimated to be of great value. To get acquainted to many colleagues from all the Nordic countries is seen as most valuable for the future.

Mobility stipends for doctoral students
In 2004 we had three mobility stipends and the same number in 2005. For example Monica Johansson from Luleå University of Technology stayed for a month in Uppsala, Andreas Ryve from Mälardalen University stayed for a month in Kristiansand as did Kirsti Nordström (now Hemmi) from Stockholm University. They have been careful to keep the costs to a minimum and the amount of money that the board set aside in the budget for this purpose has not been fully used. All the students who have received these stipends assure us how useful the mobility stipend has been to their research study. In 2006 two students made inquiries about mobility stipends but in the end they could not realise their plans. We hope that more students will apply for mobility stipends in coming years and we do not want to lower the amount marked for this purpose in the budget. It might take some time for students to understand how the option can be used.

Support for travel and participation in doctoral courses
We have been able to support all students who asked for travel support. As the knowledge and awareness about the possibility of support from NoGSME is spreading, we have seen the costs for travels to rise. Students are only requested to send in an application with an estimate of expected cost and they will receive a response from NoGSME indicating the support available. Students then submit a claim for their travel expenditure with receipts and receive the reimbursement of all the costs incurred. The board hopes that in the future it will be possible to continue to be able to respond positively to all students' applications for covering their travel expenses. In the budget for 2007 we have made sure that there is enough money for this part of our work as this seems to be highly valuable to the students. The number of active students is increasing so that also justifies an increase in the budget for this purpose.

Seminars for supervisors
In the application for a graduate school, we pointed to the fact that finding good supervisors in mathematics education is challenging in the graduate education in most Nordic countries. Thus, activities designed to build the competence of supervisors are important. We started in 2004 with a seminar in Vasa for supervisors, where we investigated quality in scientific papers. Professors Frank Lester and Diana Lambdin were invited lecturers. In this seminar, we also discussed with supervisors what they would want for the future. The outcome has guided our further activities. Thus, a second seminar took place in Korsör in April 2005 with about 25 participating supervisors, and here the theme was quality of dissertations and the work of an opponent. Presentations of dissertations were made by professors Mogens Niss, Carl Winslöw, Erkki Pehkonen and Gunnar Gjone. The third seminar took place in

Trondheim in September 1-2, 2005 just before the fourth Nordic conference in mathematics education, NORMA05. In this, focus was on the supervision process and a very experienced supervisor, Professor Uri Leron form Israel was invited to lead the seminar. Finally, in November, 14-15, 2005 a fourth seminar took place in Lund in Sweden with 20 supervisors under the guidance of professor Ole Björkqvist. The aim of this seminar was to experience reviewing of scientific papers. Authentic papers were used, and participants in groups wrote their review reports together. It was a productive learning experience for all and especially useful for supervisors who aim to support their doctoral students in submitting papers to scientific journals. As the seminars and workshops have been so well received, we plan to continue with similar activities also for 2006 and beyond. This activity is proving successful in building supervisor competence for the future and we have tried to introduce new young researchers in the group. The seminars were highly appreciated by the supervisors so in 2006 there has been another two seminars for supervisors. The fifth seminar took place in Vasa, Finland in May 5-6 2006 and the theme was research education programmes in mathematics education. The sixth seminar took place in Magleås in Denmark in October 5-6 2006. Here we discussed the preparation of good supervisors in mathematics education and many of the new doctors in the field participated. The relation between supervisor and doctoral student was in focus and we used work by Professor Jitka Linden with crucial episodes in supervision to illustrate this part.
Seminar number seven took place in Trondheim in February 8-9, 2007 and work was focussed on the review process of papers for scientific journals. It was organised in collaboration with NSMO and Nomad. We worked with authentic papers and reviews of them and discussed what characterises a good review and how to make good use of reviewers' comments.

Workshops on specific research areas
NoGSME has organised four workshops on specific research areas so far. The first one took place during the Norma05 conference in Trondheim in September 2005 and was about Classroom research in mathematics education. Simon Goodchild was invited to lead that workshop. It has been documented in the proceedings from the conference (Goodchild, 2007). The second workshop concerned research on textbooks in mathematics and took place in Kristiansand in May 2006. It had about 20 participants (both students and supervisors). Invited speakers were Birgit Pepin and Linda Haggarty.
The third workshop, also in Kristiansand, in November 2006 was about research in use of ICT in mathematics education and had about 25 participants. Invited lecturers were Luc Trouche and John Monaghan. Finally the most recent workshop was in April 2007 in Uppsala and had 27 participants. It dealt with research on mathematics and language and the speakers were Heinz Steinbring and Candia Morgan.
With the mixture of doctoral students and more experienced researchers the workshops offer a learning environment for all and an enculturation of new researchers into the field. All participants contribute with a presentation and are invited to send their contribution to the documentation. The presentations in the form of Power Point presentations are immediately shared among all.

Visibility of NoGSME
In order to make NoGSME known and visible the board has taken a number of steps. A web page is available at www.nogsme.no and a brochure (available at the web page) about

NoGSME has been produced and is used to disseminate at meetings, courses and conferences. The director and board members have presented NoGSME at different occasions (a Power point presentation has been produced). NoGSME is announced on the web pages of Agder University College. In each issue of Nomad there are 2-4 pages about the NoGSME activities and a number of papers have been published related to NoGSME (see list of publications below).

Description of current status of research and researcher training in each of the Nordic countries

A summary of the situation in 2003 as we described it in the application:
"Mathematics education is a multi-disciplinary and expanding subject in the Nordic countries. Recently, research in mathematics education has developed remarkably and in a very fast way in the Nordic countries. In the past, research in areas of mathematics education has often been undertaken in the contexts of disciplinary pedagogy or mathematics with an obvious difficulty because a multi-disciplinary subject is not easily handled inside only one disciplinary context. There is a need to give research in mathematics education and education of researchers in this field in its own right. This has happened in Finland about ten years ago through the Graduate School in mathematics and science education, in Sweden three years ago through the Graduate School in mathematics with a direction of subject didactics and in Denmark this year through a Graduate school in mathematics and science education and also in some individual departments. In 2002 in Agder University College (AUC) in Norway, the department of mathematical subjects was given the right to build a programme for education of researchers in mathematics education (matematikkdidaktik). Three new professors were employed and one lecturer already employed at the department was promoted to professor. The department already had a professor in the history of mathematics and several senior lecturers. A successful master education in mathematics education has been going on at AUC since 1994 (more than 50 master degrees awarded). During its first year, five doctoral students have been studying in the doctoral program. Another four or five doctoral students will start in September 2003. Agder University College is working actively to build a high quality programme for the education of researchers. In this endeavour we are willing to be responsible for the Nordic Graduate School in Mathematics Education. In the planning phase of this application, we have met with strong support for the idea of a Nordic Graduate School in mathematics education. Especially small environments, which have not yet been able to build a full research education programme in mathematics education, are eager to participate and to get support from a Graduate School. It is obvious that a Nordic Graduate School in mathematics education can enable synergy effects, create quality in research and cooperation across the borders. We can build on a long-standing collaboration in the Nordic countries in other endeavours concerning mathematics education".

That was the situation in 2003 and what has changed since then? The Danish and Finnish graduate Schools are still working but the Swedish was a time-limited creation for the period 2001-2006 and has now closed down. Eight of the 21 Swedish students that were taken up in 2001 in the five year long doctoral education finished their degree so far. Three more are expected to finish soon. The programme in AUC has grown and has 20 doctoral students from all over Norway at the moment. Two of the students have defended their theses so far, but another five are very near the end of their work. New doctoral stipends are announced regularly. The Danish Graduate School has only few students in mathematics education and few stipends to offer. The Finnish is greater, though. The departments that were part of the Graduate School in Sweden have only offered two stipends on their own outside the graduate school so far. A new application for a Swedish Graduate School has been sent to the Swedish Research Council for the third time in April 2007. The Baltic institutions have very few stipends to offer to students in mathematics education although there are traditions for research in mathematics education in several universities. Thus they are especially grateful for the offers they can get through NoGSME to enforce the limited

activity of research education so far. A crucial issue for the future of mathematics education research is access to stipends for financing the doctoral education of students.
Since the start of the Nordic Graduate School more institutions have entered the work and more supervisors are enrolled (about 110). The number of doctoral students was about 85 at most but in 2006 the top number of 21 students took their degree and thus ended their time as doctoral student. NoGSME has now enrolled them as prospective supervisors and cared for seminar programmes suitable for their education as supervisors. As can be seen from the list of students and supervisors above many of the research environments are very limited with only two or three persons in the area.
How the strength of the Nordic community in mathematics education has grown over the latest years became very visible during the NORMA05-conference in Trondheim. Among the contributors to this conference we find many of the doctoral students in NoGSME and also many of the supervisors. This fourth Nordic conference in mathematic education produced proceedings after a serious review process (for the first time in these conferences) and resulted in a rich book that gives good overview of the research going on in the Nordic countries (Bergsten, Grevholm, Måsöval & Rönning, 2007).
Another striking fact illustrating the viability of the Nordic community in research in mathematics education is the impressive number of theses that were defended during 2006. We know of at least 21 doctoral dissertations in mathematics education that took place this year and that must be the highest number ever. For more information see Appendix 1.
The cooperation with the Baltic countries has been good from the start of the Nordic Graduate School and we see a growing number of doctoral students taking part in summer schools and of supervisors taking part in seminars and workshops. On May 12, 2007 there will be a NoGSME seminar for Baltic supervisors in Riga, just after the eight international conference that takes part there during May 10-11. This is based on a suggestion from Madis Lepik and Janis Mencis both working in Estonia and Latvia, respectively. The focus will be on questions concerning publication in scientific journals, for example Nomad.
Over the years NoGSME has established a tight collaboration with Nomad (Nordic Studies in Mathematics Education) and many of the supervisors are now acting as referees and we see a growing number of papers from the doctoral students in NoGSME published in Nomad. Nomad had a difficult period during 2001-2004 but has now a sufficient number of papers coming in and a review process managed by the editors in such a way that the four issues have been coming out in good order since 2004. This stimulating cooperation between NoGSME and Nomad is giving good synergy effect for both activities. Another healthy sign for the field is the fact that all the new doctors have been offered important and responsible positions at once when they finish their degree. In each issue of Nomad the director of NoGSME has written a report on prospective activities (Grevholm, 2004-2006, se list of publications).
The cooperation to date with the Nordic countries and Baltic states has a strong and important impact on the Nordic Graduate School and obviously a stimulating effect on research training in all of the area.

Connections to relevant national initiatives (max ½ page)
NoGME collaborates as mentioned above with the national graduate schools in Denmark and Finland and with the Norwegian research education at AUC (which serves as a national graduate school. NoGSME also has a close cooperation with the two national centres for mathematics learning, NSMO in Trondheim Norway lead by Ingvill Stedöy-Johansen, one

of the supervisors in NoGSME, and with NCM in Gothenburg, Sweden lead by Bengt Johansson. The mathematics education specialist library in NCM and the librarian at NCM are valuable resources to the doctoral students and supervisors.
NSMO did already in the letter of intent to NoGSME promise to manage an open database over persons, projects and institutions in mathematics education and both part have been working hard to get this database going. There are still a few features in the delivered product that do not fit to the description in the order of the database, but we continue to work on it. This database is intended to live also when NoGSME has finished its funded time.
NoGSME is also collaborating with the Swedish Society of Research in Mathematics Education (SMDF), the Finnish society for Research in Science and Mathematics and Forum, the Danish society for the same purpose. For example, the workshop in Uppsala was organised in collaboration between SMDF and NoGSME. The director was in 2005 invited to give plenary lecture at one of the conferences by the Finnish Society.
The director has also given invited talks on the NoGSME activities in the Chech Republic, in Nitra University in Slovakia, in Tallinn and Tartu Universities and in some Swedish conferences.

Description of the international collaboration (max ½ page)
International collaborators from five centres of excellence supported NoGSME with letters of intent in the application and have served as mentors during our work. These are Professor Gilah Leder, Director of the Institute of Advanced Study, La Trobe University, Melbourne, Professor Anna Sierpinska, Concordia University, Montreal, Canada, Professors Hyman Bass and Deborah Ball, University of Michigan, US, Professor Michele Artigue, University Paris 7 and Director of the Intstitute of Research in Mathematics Education, and Professor Willibald Dörfler, Abteilung fur Didaktik der Mathematik, Institut fur Mathematik, Klagenfurt, Austria.
Our international contacts have been of great value in for example creating opportunities for doctoral students to arrange a stay abroad for a specific period of time, acting as additional supervisors, giving lectures at the Nordic conferences and meeting, suggesting appropriate journals for students to send papers to, and to go for conferences, acting as group leader in a summer school, collaborating with local departments and the national graduate schools and many other things.
In addition to the mentioned mentors above NoGSME has also accepted support from many other international scholars as contributors in the seminars, workshop and summer schools. Many of these persons are named in the description above of the activities. These contacts have been highly valuable for the doctoral students and supervisors and helpful in creating a satisfactory quality according to international standards of the doctoral work of students.

Description of the organisation: leadership, coordination, division of responsibilities, monitoring (max 1 page)

In the application to NorFA in 2003 we wrote:
The organisation of the Nordic Graduate School in Mathematics Education
The Graduate School will have a board consisting of a director and one member from each of the Nordic countries.

The persons suggested to be in the board (in attachment d) below) have all been asked and have agreed to be
The director will be responsible for the continuous work, preparation of the meetings of the board, for the administration and for carrying out the decisions taken in the board.
The board will decide about the plan for activities, the budget and report of the results for the economy and the work. The board (through the director) will give as tasks to groups or researchers participating in the graduate school to prepare and carry out the activities. The board will be responsible for the scientific quality, documentation and evaluation of the activities.
The distribution of tasks to departments in the Nordic countries will be well prepared thus making best possible use of the competence and opportunities available.
The director will be situated at the Agder University College, as the initiator of the Nordic Graduate School and having in one place most professors working in mathematics education in the Nordic countries. Agder University College will supply needed assistance for administration through persons working there.
The economy of The Nordic Graduate School will be handled by the Department of mathematical sciences at Agder University College and reported yearly to NorFa.
The board will have meetings twice a year to plan the activities. Decision about the budget will be taken during November-December each year. The board will try to plan the meetings in connection to courses or seminars/workshops in order to keep the costs for board meetings as low as possible.
The board will seek additional funding for costs, which can be supported from other sources.

This is an adequate description of how NoGSME was organised during 2004 to 2007. Two of the original board members had to leave for personal reasons (Rudolf Strässer and Gudmundur Birgirsson) and were substituted by members from the same countries, Christer Bergsten and Gudny Gudmundsdottir. Many participating institutions have accepted to contribute in organising the seminars and workshops so that hey have been evenly spread over all countries (except for Iceland because of the cost for travel). Different departments have assisted in organising the summer schools, also in different countries. Working in this way we hope that the activities can continue after the funding has finished as we are creating traditions for cooperation and sharing of responsibility and opening up activities for persons outside the local environment. Those who are doctoral students now get used to work in this wider community, find it rewarding and in some years they will be in the leading positions and then hopefully finding is useful to continue this work in the Nordic community they will follow up the traditions created.
International scholars visiting NoGSME in our activities report that they find the atmosphere supportive and rewarding and find that we succeeded in building a learning community that is inclusive and open.
The board has had, as in 2005, three meetings during 2006, in January, May and October. In 2007 there was a board meeting in February with planning work for the whole year. The board meetings have been scheduled in connection to other arrangements by NoGSME and thus it has been possible to keep the costs down. Between the meetings members of the board communicate via email and telephone. During the summer school 2006 there was also an informal board meeting as many board members were present there. To try to keep costs down enables us to use as much as possible to ensure maximum support for the activities that directly benefit for students and supervisors.

Description of process of recruiting students (max ½ page)

The doctoral students are recruited by each local institution that has got a scholarship free. The institution follows the regulations for their research education. NoGSME can help in informing about stipends of scholarships available but is not involved in the application of students and acceptances to the doctoral programmes. NoGSME get many requests from

international students and can direct them to an appropriate institution. The demand for stipends is higher than the available number of stipends.

Description of the plans for continuing the cooperation after the research school period (exit plan) (max ½ page)

The board has started to discuss this issue and will continue to do so. A suggestion to create a Nordic Society for Research in Mathematics Education was put forward by the director and discussed in the Norma05-conference. Work is going on with this idea and the result will be presented during Norma08 in Denmark. Such an organisation could take the role of the NoGSME board and care for Nordic initiatives of collaboration. The actual work can then be shared among the participating institutions in a similar way as we have done during the life of NoGSME. This organisation can also be a home for Nomad and for the management of the Norma-conferences, which up to now has just taken place because of personal initiatives in different countries. The board will continue to work for the survival of the NoGSME-spirit. The way NoGSME has functioned so far would be easy to carry further in a Nordic Society.

Description of the use of finances (max 1 page)

The board of NoGSME has been careful to use the funding according to the aims that were set up in the application. Very little has been used on administration and coordination of the work and much for the support of doctoral students and supervisors. As we got a flying start on January 2004, with the decision taken just a short time before that, some arrangements could not be carried out fully in 2004, as for example the summer school. Much work in the beginning went to establish communication with all supervisors and students. Once that was in place we were able of run the whole programme and the activities have become more and more each year. The costs for 2006 were close to one Million, so very near budget estimations. This year the summer school in Iceland will be very expensive so we can make good use of some of the money we were allowed to keep from earlier years. The different activities cost about as much as we have estimated in the budget. The post where we did not spend all money according to budget are mobility stipends. As we see them as important we did not want to take away this option so far. We always try to get as much as possible out of the money for students and supervisors. The accountant at the department at AUC is pleased with the way we have handled budget and costs.

List of relevant student output for the period (papers, presentations) as Appendix

In Appendix 1 we enclose short descriptions of the 21 dissertations that were finished during 2006 as an example of student output. We also refer to recent issues of Nomad where many of the students have published and to the Norma05 conference book (Bergsten, Grevholm, Måsöval & Rönning, 2007), where 18 of the doctoral students in NoGSME and 14 of the supervisors have published papers (see summaries in Grevholm, 2007). Some students also published in ESM, JRME, MERJ and JMTE and other international journals.

3 Reflections on the experiences **(max 3 pages)**

Success factors and specific problems encountered during the planning process and implementation of the research school in terms of

- objectives and strategy for the research school
- impact of the research school on research training in the field
- connections to national initiatives and international cooperation
- the need for researchers in the field and future prospective
- activities during the period and concrete results
- supervising and student activities
- the organisation of the research school
- the functioning of the Nordic partnership
- visibility and Nordic identity of the research school
- added value of being a Nordic research school
- the finances available and their use
- any other aspects relevant to you

Both doctoral students and supervisors witness to the board of NoGSME how useful the existence and activities of NoGSME are to them. Many of the groups at different universities are very small and it is crucial to meet with colleagues and discuss matters concerning the research. NoGSME can offer windows to other colleagues and other experiences than the home institution can offer. We know that access to experienced supervisors is a needle's eye for research training. Through the competence development of supervisors NoGSME has contributed to building future strength. The board appreciates the value of the work we can do thanks to the funding from NordForsk and the value it has to the Nordic community in mathematics education.
So what have the specific problems been? It is not easy to establish a functioning communication with 110 supervisors and 85 students. We have struggled a lot with email lists. The updating of lists takes much time and is not running by itself. People forget to tell us that they have moved or that they have graduated or that new students have been taken up in a programme. One can reflect on the proportion of work that goes to such rather simple things.
Another time consuming part of the work for the director has been all the applications for travel money and then the following travel bills to check and send further. But these are practical things that must follow the regulations and can not be avoided.
Our tries to get a database going in collaboration with NSMO has not been successful so far and it is disappointing to the board. But we will not give up, on the contrary we continue to work on this issue as it is one part of the options for the survival of the NoGSME-spirit after finished funding. As NSMO (which does not have a limited life) will support and run the database it would give an opportunity for prolonged collaboration in the Nordic community.
Reflecting on these small practical problems it is then obvious that the scientific part of the work has been rewarding. It has not been difficult to find ideas for programme for seminars and workshops. When we wanted to invite international scholar to our work they have always been willing to come and contributed in very good ways. Thus the activities have

been highly appreciated and the board considers them valuable for the future of mathematics education in the Nordic countries. The institutions we have approached asking them for collaboration on a specific practical arrangement have always been willing to contribute and done good work. Initiatives for new courses have been carried out in the best spirit of cooperation between the board and the hosting department.
For the students the most important thing beside what they learn in the courses and summer schools seems to be that they become friends with many other prospective scholars in the field. They know much about each others research projects and areas of interest. One can imagine in ten years time when all these doctors in mathematics education are sitting on important positions around in the Nordic countries and other places how important it is to them to be able to contact a colleague in NoGSME, whom they know since long and where they can ask for different kinds of support, experiences or insights. It is obvious that we are building strength for the future when all these doctoral students are offered opportunities to work together and get acquainted.
In the summer school students spend a whole week together in a working group lead by an experienced scholar. This is a unique situation and the deep reflections on each others work that they are able to do during this period are very influential for them. The fact that others show interest of one student's work and deal with it in serious ways for a longer period has much value. Many of the participants have expressed just that kind of thoughts.
The need for researchers in the field still seems to be high. As mentioned above all the new doctors so far have got an important and responsible position at once after graduation. The positions are not only in universities but also in official authorities, communities and schools. In all the Nordic countries recent evaluations show that there is a need for more teachers with a doctoral degree in teacher education. We have also the fact that most of the professors in mathematics education and many of the university lecturers in the Nordic countries are approaching retirement age and will thus need successors rather soon.
The board of NoGSME considers the decision of NordForsk to fund the Nordic Graduate School of Mathematics Education to be of utmost value to the research community in mathematics education in the Nordic countries. We foresee that the value of this investment will be even more visible in the future.

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Appendix 1
Theses during the year 2006 – a rich year in mathematics education

Barbro Grevholm, Agder University College

In SMDF Newsletter we have tried to present regularly short presentations of Swedish theses in mathematics education, as they have come to our knowledge. During the year 2006 this task has grown substantially as we have found at least twenty one dissertations in mathematics education in the Nordic countries. Twelve of them are from Sweden and from them eight are from the Swedish Graduate School. This high level of production will probably not be seen again for some time. One contributing factor is of course that all the 21 doctoral students in the Swedish Graduate School started in the same time and were supposed to finish during 2006. The short presentations are intended to be appetizers and we hope that your curiosity will wake up and take you to a closer reading of the theses. Such a short paragraph as we give here can of course not do justice to work that has taken several years and comprise several hundreds of pages. Thus we recommend you to investigate the theses further.

Morten Misfeldt defended his dissertation in January at the Danish University of Education. The title is Mathematical writing and it consist of a monograph, where all chapters except two build on earlier published work or work in progress (17 titles), such as one journal paper, 5 reviewed conference presentations, 3 book chapters and some other publications. The subject of the thesis is *Mathematical writing* and how various technologies support such an activity. It consists if an empirical and a theoretical part. One of the main theoretical results is the attempt to view that mathematical writing process simultaneously as a creative writing process and as a mathematical problem-solving process.

Kristin Bjarnardottir defended her thesis at Roskilde University Center in February. Her thesis carries the title *Mathematical Education in Iceland in a Historical context of socio-economic demands and influences*. Her data consists of historical documents from the development of school mathematics in Iceland and her dissertation is a monograph. The thesis consists of three parts and examines the history of mathematical education in Iceland and its position in comparison to its neighbouring countries. Mathematics education in Iceland differs primarily form its neighbours in the absence of demand for furthering higher mathematical education, nearly total dominance of a few institutions, and initiatives of individuals.

In March Iiris Attorps defended her dissertation *Mathematics teachers' conceptions about equations* at the University of Helsinki. Her work is a monograph (a thick one with 231 pages). The aim of the study is to describe and clarify the mathematics teachers' subject matter and pedagogical content conceptions about equations. The research results show that some of the teachers do not have clear conception of what the pupils should attain in algebra at compulsory school. Both expert and novice teachers have various apprehensions of the pupils' difficulties concerning equations.

Kristina Juter defended her dissertation in the beginning of April at Kristianstad University. The thesis comprises six published papers and an extended summary binding them together (referred 'kappa' in Swedish). The title of her work is *Limits of functions – University students' concept development.* She followed university students during their first course of tertiary mathematics and inquired into their development of the limit concept. One of her results is that many students complete basic courses in calculus without ever understanding the notion of limits. Kristina Juter interprets her findings to argue that connectedness and continuity are essential features of teaching and learning limits to prevent students from failing.

Lil Engström carried out her defence later in April and the work is called *Möjligheter till lärande i matematik* (Opportunities to learn mathematics). It is a monograph, written in Swedish and as Attorps' a thick one (238 pages). The study examines how teachers formulate mathematical problems, how they use the experiences students have gained and what use they make of the software's potential. One result is that the teachers' ability to pose thought-provoking open-ended problems is the most important factor as it significantly influences what the students learn.

Mette Andresen defended her dissertation on *Taking advantage of computer use for increased flexibility of mathematical conceptions* in Mid May at the Danish University of Pedagogy. Mette Andresen's study is part of a larger project in Denmark called World Class Math & Science. In a sub project each student had a laptop at her disposal with computer algebra software. Participants gained the experience that computer use in upper secondary school mathematics has a potential. Mette studied: "How could these potentials be captured and conceptualised?" Her later question became: "Is flexibility a supportive construct for articulation of experiences of teaching and learning within a modelling approach? Is it useful for realisation of the learning potential of students' concept formation?" The methods can be described as an interpretative approach teaching experiment design with classroom observations in four schools, video recordings and field notes.

Gunnar Sjöberg defended his dissertation at Umeå University in the Graduate School of Pedagogical work. The title is Om det inte är dyskalkyli – vad är det då? (If it is not dyscalculia – what is it then?). He has investigated the concept of dyscalculia in the research literature and finds that it is an ill-defined or not defined concept. The pupils he has followed from year five in school to upper secondary school were said to be in mathematics problems but many of them succeded later in the subject. One crucial factor seems to be the short time these pupils spend on mathematics learning, often less that half an hour per school week. Compulsory school mathematics teachers will find interesting interviews in this work, with pupils' comments that have much to say to them.

Monica Johansson wrote about *Teaching mathematics with textbooks – A classroom and curricular perspective*. She defended her work in June at Luleå University of Technology. The dissertation consists of four papers and a preamble and the focus of all the parts is the relationship between the textbook and the curriculum. She shows that the textbook influences not only what kind of tasks students are working with during the lessons, but also the examples the teacher presents on the board, what kind of concepts are introduced

and how they are introduced. The teacher can get into problems because of too much reliance on the textbook. The study shows the relative autonomy of the mathematics teacher in relation to the most common teaching tool in Swedish classrooms - the textbook.

Örjan Hansson is also a member of the Swedish Graduate School in Mathematics Education, as is Monica, and he defended his dissertation the day after Monica at the same university. The work carries the title *Studying the views of pre-service teachers on the concept of function.* His work consists of five papers and an overview that binds the work together. Three different groups of pre-service teachers for year 4-9 in mathematics and science have been his informants. He has used questionnaires, concept maps, and interviews in order to understand and analyse how they perceive the concept of function. The concept of function is rarely a well integrated concept and the pre-service teachers view of the concept is represented by a less developed knowledge structure than on could wish for a prospective teacher. Thus there are many implications for the teaching of pre-service teachers.

On the same day as Örjan, Maria Bjerneby-Häll defended her thesis at Linköping University with the title: *Allt har förändrats och allt är sig likt: En longitudinell studie av argument för grundskolans matematikundervisning.* The aim of this thesis is to describe and analyse arguments for mathematics in compulsory school and to understand why and how the official arguments change. The point of departure is that the conditions and the reality for school mathematics can be understood through an analysis of official arguments and of personal arguments given by teacher students and teachers. A group of teacher students has been followed during their education and the first three years after their professional debut. The result shows that during their education the teacher students develop a view on mathematics and mathematics education harmonizing with the goals of mathematics in the national syllabus. The novice teachers experience quite different conditions when they start to work as teachers. Preparing the pupils for the national test becomes the most important goal for the novice teachers. A factor influencing the mathematics teacher is the qualification requirement in mathematics from compulsory school to go into the national programs in the upper secondary school. The novice teachers experience a conflict between different goals in the national curriculum and course syllabus for mathematics.

Andreas Ryve's dissertation took place at Mälardalen University in the end of June with Anna Sfard as opponent. The title is *Approching mathematical discourse. Two analytical frameworks and their relation to problem solving interactions*. His main aim of the study is to investigate how conceptual understanding and problem solving can become a natural part of mathematics teaching and thus of students' mathematical knowledge construction. He wants to characterize the classroom discourse in two different problem solving courses in teacher education and also to investigate and further develop two analytical frameworks – a communicational approach and a dialogical approach used to study mathematical discourses. He shows that the classroom discourse can be characterized in terms of subject oriented, didactically oriented and problem solving oriented discourses. The analytical frameworks are further developed in his study.

At Ålborg University Lene Östergaard Johansen defended her thesis "Hvorfor skal voksne tilbydes undervisning i matematik? – en diskursanalytisk tilgang til begrundelsesproblemet" (Why should adults be offered teaching in mathematics? – a discourse-analytical approach to the problem of justification). She claims that justification for mathematics teaching is rarely explicit and only on specific events can we get access to the real reasons for why a decided group of students are offered mathematics. She has chosen to answer the question by analysing the development of preparatory mathematics teaching for adults (FVU), which was introduced by a Danish reform in 1999. Lene differs analytically between three discourses: The political discourse, The planning of teaching persons' discourse and The mathematics teachers' discourse. In her thesis she develops a framework for analysing the explicit and implicit justifications. Although there are common explicit justifications in the system, her analyses show that there are conflicting implicit justifications. These conflicting justifications build upon different view of human beings, different views of mathematics knowledge and skills and of mathematics learning.

Kirsti Hemmi at Stockholm University defended her thesis "Approaching proof in a community of mathematical practice". Her aim is to describe how students encounter proof in a community of mathematical practice at a mathematics department and how they are drawn to share mathematicians' views and knowledge of proof. She tries to combine socio-cultural theories, social practice theories and theories about proof into a framework for understanding and describing the diversity of the culture which involves the complex notion of proof. Students felt that they were confronted with proofs from the beginning of their studies. Proof was there as a mysterious artefact and many aspects of proof remained invisible for the students when they struggled to find out what a proof is and to understand its role and meaning in the practice. The first oral examination in proof seems to be significant in drawing students to the practice of proof.

At Umeå University Jesper Boesen defended his dissertation "Assessing mathematical creativity." The thesis consists of four papers and a preamble. Jesper claims that use of superficial reasoning seems to be a main reason for learning difficulties in mathematics. Therefore he finds it important to investigate reasons for this use and the components that may affect students' mathematical reasoning development. Assessments have been claimed to be one such component that significantly influence students' learning. The study shows that a majority of the tasks in the teacher-made assessment could be solved by successfully using only imitative reasoning. The national tests however required creative mathematically founded reasoning to a much higher extent. He also investigates what kind of reasoning the students really use, why teacher made tests emphasise low-quality reasoning and if national tests influence teachers in their development of classroom tests. This impact seems to have a limited effect.

Ole Einar Torkildsen defended his thesis for the academic degree of doctor philos at Oslo University. It has the title *Mathematical archaeology on pupils' mathematical texts. Un-earthing of mathematical structures*. The data basis for his work is the pupils' solutions for six tasks given in a competition in Tangenten in the early 1990's. They make up 23 mathematical texts. The purpose of his analysis method is to make explicit the mathematical structures that are inherent in the pupils' solutions. Thus he is intending to uncover the mathematical structures by analysing pupils' solutions through the glasses of a

mathematician. The analysis revealed that the pupils in their solving process heavily relied on some fundamental mathematical structures. The relationships in the answers can be identified as functions, in some instances functions with more than one independent variable. The mathematical structures are localised at two levels and factors influencing the solution process are studied. Finally Torkildsen argues that mathematical archaeology is a suitable tool for increasing the knowledge about pupils' mathematical activity.

Åse Streitlien's thesis has the title *Room for participation – a study of interaction and communication in mathematics classrooms*. It is written in Norwegian and makes up an extensive text of 350 pages and was defended at Oslo University. The aim of her work is to study interaction and communication between the teacher and her students in mathematics classrooms. The research focus is concerned with the opportunities young students have for participating in the discourse of mathematics and how the dynamics of reasoning and discussion gives rise to mathematical meaning. Taped lessons from two classrooms were analysed using discourse analysis. Focus was on how discourse patterns influence what counts as mathematics knowledge and what communicative competences the students need for participating in the classroom discourse. Streitlien suggests that what students learn in mathematics depends on how their teacher responds to their responses and the opportunities there are given them in the negotiation of mathematical meaning.

Ewa Bergqvist calls her thesis *Mathematics and mathematics education – two sides of the same coin: some results on positive currents related to polynomial convexity and creative reasoning in university exams in mathematics.* It was defended at Umeå University in Sweden. The dissertation consists of two different but connected parts. Part A is based on two papers in mathematics and part B on two papers in mathematics didactics. In part B focus is on what kind of reasoning university students in mathematics use in courses and exams. Bergqvist differs between imitative reasoning and creative reasoning. About 70 % of the tasks in exams can be solved by imitative reasoning. The teachers constructing the exams are pleased with this situation. They claim that otherwise the exams would be too difficult and lead to too low passing rates.

Sharada Gade was the first to present her thesis in the doctoral programme of Agder University College in Norway. The title is *The micro-culture of a mathematics classroom. Artefacts and activity in meaning making and problem solving*. The work is based on a yearlong classroom study in the first grade in upper secondary school in Norway. The thesis points to the centrality of both meaning-making in teaching-learning and goal-directedness in problem-solving, as important parts of the instruction in a mathematics classroom. The classroom was bilingual with emphasis on cooperation or group-learning by students. The thesis offers a synthesis based on socio-cultural perspectives of the micro-culture of teaching-learning of mathematics established and situated in the classroom.

Magnus Österholm defended his thesis at Linköping University and it has the title *Cognitive and metacognitive perspectives on reading comprehension in mathematics*. The purpose of the dissertation is to examine whether a reader needs special types of knowledge or abilities in order to read mathematical texts. The reading of mathematical texts is studied from a cognitive perspective and from a meta-cognitive perspective. In the first case reading abilities and content knowledge are studied in relation to reading comprehension.

In the second case the focus is on beliefs and how a reader determines whether a text has been understood or not. The results show that courses at upper secondary level and at university level do not affect the special reading ability. There is a need to focus on reading but it does not need to be about learning to read mathematical texts but to use existing, more general reading ability also for mathematical texts.

Markus Hähkiöniemi at the University of Jyväskylä has written and defended the thesis *The role of representations in learning the derivative*. The aim of the study is to find out how students may use different kinds of representations for thinking about the derivative in a specific approach. To achieve this, the author designed and implemented a five-hour teaching-learning sequence introducing the derivative concept in a Finnish high school (grade 11). Five students were selected to take part in carefully designed task-based interviews. He found that the embodied world offered powerful thinking tools for the students. They used increase, steepness, horizontalness and tangent of the graph for thinking about the derivative qualitatively without calculating anything. On the basis of the analysis of the students' use of representations a hypothetical learning path to the derivative was constructed.

Per Nilsson's thesis presented at Växjö University has the title *Exploring probabilistic reasoning. A study of how students contextualise compound chance encounters in explorative settings.* The focus is on what learners with little experience of formal theories of probability do and can do then they are dealing with compound random situations in which they are offered opportunities to integrate different probabilistic lines of reasoning. Two part studies have been done on 12 to 13 year old students and one study on 14 to 16 year old students. The younger students acted within a dice-game setting and the older with ICT-versions of compound, independent events. Prior to instruction students were able to devise ideas of underlying probability distributions in the case of compound random phenomena. The students brought into the discussions geometrical and numerical considerations as well as arguments reflecting principles of the law of large numbers.

What will all this new knowledge about teaching and learning mathematics result in? Will we find changes in the mathematics classrooms as a consequence of the work presented? Hopefully we will but we must be aware that it takes time to change such heavy systems as school and the educational system. It would be interesting in the future to inquire into the effect of these dissertations on mathematics teaching and learning in the Nordic countries. Maybe some prospective doctoral student will take this challenge?

Appendix 2

Course literature for the courses given at AUC

Course literature MA-601 (autumn 2006)

(This will be updated and edited each time the course is given)

Breiteig, T (2006)(Ed.). *Theory of Science from a perspective of Mathematics Education. MA-601 – A reader*. Kristiansand: Agder University College.
(Available from SørBok).

Ernest, P. (1991). *The Philosophy of Mathematics Education*. London: Falmer Press.

Synopsis

Although many agree that all teaching rests on a theory of knowledge, there has been no in-depth exploration of the implications of the philosophy of mathematics for education. This is Paul Ernest's aim. Building on the work of Lakatos and Wittgenstein it challenges the prevalent notion that mathematical knowledge is certain, absolute and neutral, and offers instead an account of mathematics as a social construction. This has profound educational implications for social issues, including gender, race and multiculturalism; for pedagogy, including investigations and problem solving; and challenges hierarchical views of mathematics, learning and ability. Beyond this, the book offers a well-grounded model of five educational ideologies, each with its own epistemology, values, aims and social group of adherents. An analysis of the impact of these groups on the National Curriculum results in a powerful critique, revealing the questionable assumptions, values and interests upon which it rests. The book finishes on an optimistic note, arguing that pedagogy, left unspecified by the National Curriculum, is the way to achieve the radical aims of educating confident problem posers and solvers who are able to critically evaluate the social uses of mathematics.

Hersh, R. (1998). *What is mathematics really?* Vintage.

Synopsis

In *What Is Mathematics, Really?*, author Reuben Hersh proposes a philosophy of mathematics that he calls "humanism" and uses this philosophy to analyze age- old questions of proof, certainty and invention versus discovery. He also surveys the history of the philosophy of mathematics. Readers of all levels of mathematical experience will be stimulated by the fascinating and perspicacious discussions Hersh has to offer.

Niss, M. (2001). Den matematikdidaktiska forskningens karaktär och status. In B. Grevholm (ed.), *Matematikdidaktik - ett nordiskt perspektiv* (side 21-47). Lund: Studentlitteratur.
Also in B. Grevholm (2003)(ed.), *Matematikk for skolen* (pp. 335-364). Bergen: Fagbokforlaget.
Niss, M. (1999). Aspects of the nature and state of research in mathematics education. *Educational Studies in Mathematics* 40(1), 1-24.

Niss, M. (2001). Mål för matematikundervisningen. In B. Grevholm (ed.), *Matematikdidaktik - ett nordiskt perspektiv* (pp 51-90). Lund: Studentlitteratur. Also

in B. Grevholm (2003)(ed), *Matematikk for skolen* (pp. 288-334). Bergen: Fagbokforlaget.

Nissen, G. & Blomhöj, M. (1993). *Criteria for scientific quality and relevance in the didactics of mathematics education*. Roskilde: IMFUFA Roskilde University.

A selection of articles that illuminate the above mentioned areas especially will be chosen in cooperation with the course participants.

Reference literature

Generally

Davis, P. J. & Hersh, R. (1981) *The Mathematical Experience*. London: Penguin.

Ernest, Paul (ed.) *Mathematics, Education, and Philosophy. An International Perspective*; London: Falmer 1994.

Hofstadter, D.R. (2000). *Gödel, Esher, Bach: An Eternal Golden Braid* 20th anniversary edition. . London: Penguin Books

Synopsis

Linking together the music of Bach, the graphic art of Escher and the mathematical theorems of Godel, as well as ideas drawn from logic, biology, psyhcology, physics and linguistics, Hofstadter illumintaes one of the greatest mysteries of modern science: the nature of the human thought process.

Kuhn, Th. (1996). *The Structure of Scientific Revolutions*.Chicago: University of Chicago Press. (Dansk Köbenhavn 1973).

Lakatos, I., Zahar, E. & Worall, J. (Eds.)(1976). *Proofs and Refutations*. *The Logic of Scientific Discovery*. Cambridge: Cambridge University Press.

Synopsis

Proofs and Refutations is essential reading for all those interested in the methodology, the philosophy and the history of mathematics. Much of the book takes the form of a discussion between a teacher and his students. They propose various solutions to some mathematical problems and investigate the strengths and weaknesses of these solutions. Their discussion (which mirrors certain real developments in the history of mathematics) raises some philosophical problems and some problems about the nature of mathematical discovery or creativity. Imre Lakatos is concerned throughout to combat the classical picture of mathematical development as a steady accumulation of established truths. He shows that mathematics grows instead through a richer, more dramatic process of the successive improvement of creative hypotheses by attempts to ?prove? them and by criticism of these attempts: the logic of proofs and refutations.

Mellin-Olsen, S. (1987). *The politics of mathematics education*. Dordrecht: Kluwer Academic Publishers.

Niss, M. (1994). Mathematics in society. In R. Biehler et al (Eds.), *Didactics of mathematics as a scientific discipline.* Dordrecht: Kluwer Academic Publishers.

Popper, K. (2002). *The Logic of Scientific Discovery.* London: Routledge Classic.

Skovsmose, O. (1994). *Towards a Philosophy of Critical Mathematics Education*. Dordrecht, The Netherlands: Kluwer Academic Publishers.

Säljö, R. (2000). *Lärande i praktiken. Ett sociokulturellt perspektiv*. Stockholm: Prisma.

Books, articles or excerpts from books that concern special themes in the course:

What is science?

Kaiser, M. (2000). *Hva er vitenskap?* Oslo: Universitetsforlaget.

Fjelland, R. (1999). *Vitenskap mellom sikkerhet og usikkerhet.* Oslo: ad Notam Gyldendal.

Okasha, S. (2002). *Philosophy of Science. A Very Short Introduction.* Oxford: Oxford University Press.
(144 p, focus on natural science , GBP 5,60 at Amazon)

Chalmers, A.F. (1995). *Hvad er videnskab?* København: Filosofi Gyldendal.
(270 s. SørBok NOK 220. This translation from Danish is from second edition. Chalmer's book is substantially revised in the third edition.)

Chalmers, A.F. (2002). *What is this thing called Science? Third edition.* Buckingham: Open University Press.

Popper, K. Science: Conjectures and Refutations (pp. 3-10)
Kuhn. T. S. Logic of Discovery or Psychology of Research? (pp. 11-19)
Lakatos, I. Science and Pseudoscience. (pp. 20-26)
Salmon, W.C. (2001). Rationality and Objectivity in Science or Tom Kuhn Meets Tom Bayes (pp. 551-583).
All these in M. Curd & J.A. Cover (Eds.)(2001), *Philosophy of Science. The Central Issues.* London: W.W: Norton

What is mathematics?

Fosgerau, G. (1992). Hvad er matematik? In G. Fosgerau & F.H. Kristiansen (Eds.), *Midt i matematikken – En bog om matematiske spørgsmål* (pp. 21-36). Århus Fællesseminarium, Århus: Kvan.

Devlin, K. (2000). What is Mathematics? In K. Devlin, *The Language of Mathematics.* (pp. 1-12). San Fransisco: Freeman.

Hempel, C. G. (1956). Om den matematiska sanningens natur. Geometri och empirisk vetenskap. I J.Newman (Red.), *Sigma. Matematikens kulturhistoria* (vol 5, pp. 1713-1740). Stockholm: Forum.

Aleksandrov, A.D., Kolmogorov, A.N. & Lavrent'ev, M.A. (1963). A general view of Mathematics. §1 The Characteristic Feature of Mathematics. In A.D. Aleksandrov, A.N. Kolmogorov & M.A Lavrent'ev, *Mathematics its content, methods and meaning* (pp. 1-7). New York: MIT press.

Atiyah, M. (1979). Utviklingslinjer innen ren matematikk. *Normat* (1), 10-20. This is a translation of his plenary lecture at ICME-3 1976, printed in H.Athen & H. Kunle (Eds.), *Proceedings of the 3rd International Congress on Mathematical Education*. (pp 61-74). Karlsruhe: Zentralblatt für Didaktik der Mathematik.

Davis, P.J. & Hersh, R. (1986). *Descartes' Dream. The World according to Mathematics.* London: Penguin Books.

Shapiro, S. (2002). *Thinking About Mathematics: The Philosophy of Mathematics*. Oxford: Oxford University Press.

Synopsis

This text looks at a range of philosophical issues and positions concerning mathematics in four comprehensive sections. The first describes questions and issues about mathematics that have motivated philosophers almost since the beginning of intellectual history. Part II is an historical survey, discussing the role of mathematics in such thinkers as Plato, Aristotle, Kant and Mill. The third section covers the three major positions and battle lines throughout the 20th century: that mathematics is logic (logicism); that the essence of mathematics is the rule-governed manipulation of characters (formalism); and a revisionist philosophy that focuses on the mental activity of mathematics (intuitionism). Finally, Part IV looks at contemporary positions and work which brings the reader up-to-date on the discipline. The book aims to be accessible to those with little background in either mathematics or philosophy. It is aimed at students and professionals in mathematics who have little contact with academic philosophy and at philosophy students and other philosophers who forgot much of their mathematics.

Penrose, R. (1999). *The Emperor's New Mind.* Oxford: Oxford University Press.

Hanna, G. & Jahnke, H. N. (1996). Proof and proving. In A. Bishop et al (Eds.), *International handbook of mathematics education, Vol 2,(*chapter 23, 877-908). Dordrecht: Kluwer Academic Publishers.

The process of creating mathematics

Hardy, G.H. (1967). *A Mathematician's Apology.* London: Cambridge University Press. Også gjengitt på svensk i J.Newman (Red.), *Sigma. Matematikens kulturhistoria* (vol 5, pp. 2118-2133). Stockholm: Forum

Hadamard, J. (1954). *An Essay on the Psychology of Invention in the Mathematical Field*; New York: Dover Books.

Poincaré, H. (1976). Mathematical Creation. In Vernon (Ed.), *Creativity* (pp. 77-88). Harmondsworth: Penguin Books.
Also in J. Newman (Ed.), *The World of Mathematics*.
Also in Swedish in J.Newman (Red.), *Sigma. Matematikens kulturhistoria* (vol 5, pp. 2134-2145). Stockholm: Forum.

Doxiadis, A. (2001). *Onkel Petros og Goldbachs formodning.* Oslo: Pax forlag.

Mathematics education as a science. Research on the learning of mathematics

Lesh, R. & Lovitts, B. (2000). Research Agendas: Identifying Priority Problems and Developing Useful Theoretical Perspectives. In A.E. Kelly & R.A. Lesh (Eds.), *Research Design in Mathematics and Science Education* (pp. 45-71). Mahwah, New Jersey: Lawrence Erlbaum

Freudenthal, H. (1978). *Weeding and Sowing. Preface to a Science of Mathematical Education.* Dordrecht: Kluwer Academic Publisher.

Sierpinska, A.& Kilpatrick, J. (1998*). Mathematics education as a research domain: A search for identity.* Dordrecht: Kluwer Academic Publishers.

Synopsis

In 1978, in the foreword to "Weeding and Sowing": a preface to a "Science of Mathematics Education", Hans Freudenthal wrote that his book is a preface to a science that does not exist. Almost 20 years later, does his claim still hold true? The present book is the result of the reflection of individuals in mathematics education on this and related questions. Is mathematics education a science? Is it a discipline? In what sense? What is its place within other domains of research and academic disciplines? What accounts for its specificity? In the book, the reader will find a range of possible answers to these questions, a variety of analyses of the actual directions of research in different countries, and a number of visions for the future of research in mathematics education. The book is a result of an ICMI study, whose theme was formulated as: "What is Research in Mathematics Education and What are Its Results?". One important outcome of this study was the realization of the reasons for the difficulty of the questions that the study was posing, leading possibly to a set of other questions, better suited to the actual concerns and research practices of mathematics education researchers. The book addresses itself to researchers in mathematics education and all those working in their neighbourhood who are concerned with the problems of the definition of this new scientific domain emerging at their borders.

Reading List MA-602

Details of book	Code
Black, P., Harrison, C., Lee, C., Marshall, B, & Wiliam, D. (2003). Assessment for Learning: Putting it into Practice. Maidenhead: Open University Press.	3
Boaler, J. (1997) Experiencing School Mathematics: Teaching styles, sex and setting. Buckingham: Open University Press	2
Boaler, J. (2000) (Ed.) Multiple Perspectives on Mathematics Teaching and Learning. London: Ablex Publishing	3

Burton, Leone (Ed.) (1999) Learning Mathematics: From Hierarchies to Networks. London: Falmer Press.	3
Cobb, P., Yackel, E. and McClain, K. (2000). Symbolising and Communicating in Mathematics Classrooms: Perspectives on discourse, tools and instructional design. Mahwah, NJ: Lawrence Erlbaum Associates.	2
Cooper, B. and Dunne, M. (2000). Assessing Children's Mathematical Knowledge: Social class, sex and problem-solving. Buckingham: Open University Press.	2
Daniels, H. (2001). Vygotsky and Pedagogy. London: Routledge Falmer.	2
Davis, P. J. And Hersh, R. (1981) The Mathematical Experience. London: Penguin	1a
Davis, R. B., Maher, C. A. and Noddings, N. (1990) (Eds.) Constructivist Views on the Learning and Teaching of Mathematics. Journal for Research in Mathematics Education, Monograph Number 4. Reston, Va: National Council of Teachers of Mathematics.	1b
Donaldson, M. (1978) Children's Minds. London: Fontana. (Good read. Good introduction to and critique of Piaget's stage theory.)	1a
Dowling, P. (1998). The Sociology of Mathematics Education: Mathematical Myths/Pedagogic Texts. London: Falmer Press.	3
Ernest, P. (1991) The Philosophy of Mathematics Education. London: Falmer Press	1b
Fennema, E. and Scott Nelson, B. (xxxx) Teachers in Transition …	3
Floyd, A. (Ed.) (1979). Cognitive Development in the School Years. London: Croom Helm.	3
Goodchild, S. (2001) Students' Goals: a case study of activity in a mathematics classroom. Norway: Caspar Forlag	3
Grouws, D. A.and Cooney, T. J. (1988). (Eds.) Effective Mathematics Teaching. Mahwah, NJ: Lawrence Erlbaum Associates.	1b
Hughes, M. (1986). Children and Number: Difficulties in Learning Mathematics. Oxford: Blackwell	1b
Janvier, C. (1987). (Ed.) Problems of Representation in the Teaching and Learning of Mathematics. Mahwah, NJ: Lawrence Erlbaum Associates.	3
Jaworski, B. (1994). Investigating Mathematics Teaching. London: Falmer Press (especially chapters 2 on Constructivism and 4 on Research Methods)	1b
Jaworski, B. and Phillips, D. (Eds.) (1999). Comparing Standards Internationally: research and practice in mathematics and beyond. Oxford: Symposium Books.	3
Lakatos, I. (1976). Proofs and Refutations. Cambridge: Cambridge University Press.	1b
Lampert, M. (2001)Teaching Problems and the Problems of Teaching. New Haven: Yale University Press.	2
Lave, J. (1988) Cognition in Practice. Cambridge: Cambridge University Press.	3
Lave, J. and Wenger, E. (1991) Situated Learning: Legitimate Peripheral Participation. Cambridge: Cambridge University Press.	2
Maddy, P. (1990). Realism in Mathematics. Oxford: Clarendon Press.	3
Mellin-Olsen, S. (1987) The Politics of Mathematics Education. The Netherlands: Reidel	3
Nesher, P. and Kilpatrick, J. (1990) (Eds.). Mathematics and Cognition. Cambridge: Cambridge University Press.	2

Nickson, M. (2000). Teaching and Learning Mathematics: A Teacher's Guide to Recent Research and its Application. London: Cassell	1b
Nunes, T. and Bryant, P. (1996). Children Doing Mathematics. Oxford: Blackwell.	2
Pimm, D. (1988). Speaking Mathematically: Communication in Mathematics Classrooms. London: Routledge.	2 3
Restivo,S., Van Bendegem, J. P. and Fischer, R. (1993). Math Worlds: Philosophical and Social Studies of Mathematics and Mathematics Education. New York: State University of New York Press.	
Resnick, L. B. and Ford, W. W. (1984) The Psychology of Mathematics for Instruction. Mahwah, NJ: Lawrence Erlbaum Associates.	1b 1a
Richardson, K. (1985). Learning Theories. Unit 8/9 of the Open University Course: Personality, Development and Learning. Milton Keynes: The Open University.	3
Rogers, P. and Kaiser, G. (1995). (Eds.) Equity in Mathematics Education: Influences of Feminism and Culture. London: Falmer Press.	2
Rowland, T. (2000) The Pragmatics of Mathematics Education: Vagueness on Mathematical Discourse. London: Falmer Press	1b
Seeger, F., Voigt, J. and Waschescio, U. (Eds.) (1998). The culture of the mathematics classroom. Cambridge: Cambridge University Press.	3
Sierpinska, A. (1994) Understanding in Mathematics. London: Falmer Press	1b
Skemp, R. (1971). The Psychology of Learning Mathematics. London: Pelican.	2
Skemp, R. (1989). Mathematics in the Primary School. London: Routledge.	3
Skovsmose, O. (1994). Towards a Philosophy of Critical Mathematics Education. The Netherlands: Kluwer Academic Publishers	1b
Steffe, L. P. and Gale, J. (Eds.) (1995) Constructivism in Education. Hillslade, NJ: Lawrence Erlbaum	3
Steffe, L. P., Nesher, P., Cobb, P., Goldin, G. A. and Greer, B. (1996) Theories of Mathematical learning. Mahwah, NJ: Lawrence Erlbaum Associates.	2
Stigler, J. and Hiebert, J. (1999). The Teaching Gap . New York: The Free Press	2
Vygotsky, L. (1978). Mind in Society: The Development of Higher Psychological Processes. (Edited by Cole, John-Steiner, Scribner and Souberman). Cambridge, Ma: Harvard University Press.	2
Vygotsky, L. (1986). Thought and Language. (edited Kozulin) Cambridge, Ma: The MIT Press	2
Watson, A. (Ed.) (1998). Situated Cognition and the Learning of Mathematics. Oxford: Centre for Mathematics Education Research, University of Oxford Department of Educational Studies.	
Wenger, E. (1998) Communities of Practice: Learning Meaning and Identity. Cambridge: Cambridge University Press.	2
Wells, G. (1991). Dialogic Inquiry: Towards a Sociocultural Practice and Theory of Education. Cambridge: Cambridge University Press.	3
Wells, G. (Ed.) (2001). Action, Talk and Text: Learning and Teaching through Inquiry. NY: Teachers College Press.	3
Wertsch, J. V. (1985). Culture, Communication and Cognition: Vygotskian Perspectives. Cambridge: Cambridge University Press.	2

Wood, D. (1988). How Children Think and Learn. Oxford: Basil Blackwell.	
Wood, T., Scott Nelson, B. and Warfield, J. (Eds.) (2001). Beyond Classical Pedagogy: Teaching Elementary School Mathematics. Mahwah, NJ: Lawrence Erlbaum Associates.	1a 2
Special Issues/Articles etc.	
Educational Studies in Mathematics, Volume 46, Numbers 1-3, 2001. Special Issue: Bridging the Individual and the Social: Discursive Approaches to Research in Mathematics Education. Editors: Carolyn Kieran, Ellice Forma and Anna Sfard.	2
Nordisk Matematik Didaktik (NOMAD) Volume 8, Number 3, November 2000. Special Issue: Social Constructivism, Social Practice Theory and Sociocultural Theory: Relevance and Rationalisations in Mathematics Education. Editors Barbara Jaworski and Bodil Kleve.	2
***	
Bruner, J, (1997). Celebrating Divergence: Piaget and Vygotsky. Human Development 40, 63-73	
Cole, M. & Engeström, Y (1993). A cultural-historical approach to distributed cognition. In G. Salomon (Ed.), Distributed cognitions: Psychological and educational considerations. Cambridge: Cambridge University Press.	3 1a
Lerman, S. (1996). 'Intersubjectivity in Mathematics Learning: A Challenge to the Radical Constructivist Paradigm?' Journal for Research in Mathematics Education, 27(2), pp. 133-150.	2
Renshaw, P. (2002) Community and Learning. Professorial Lecture Series. Australia: Griffith University, School of Education and Professional Studies.	2
Rogoff, B. (1994) Developing Understanding of the idea of communities of learners. Mind Culture and Activity, 1, 209-229	3
Steffe, L. P. and Thompson, P. W. (2000). Interaction or Intersubjectivity? A Reply to Lerman. Journal for Research in Mathematics Education, Vol 31, No 2, 191-209.	2
If you find other articles that seem especially useful that could be added to this list, please send details to the course leader.	

MA604: Literature on problem solving

In this course you will be introduced to many references. You are not expected to read all the references. However, it is important for your essay writing that you are familiar with the problem-solving literature. The references are presented chronologically. It is our intention that the reference list could be supplemented throughout the course weeks. We have divided the list in some main categories:

1. Problem solving in a historical perspective
2. Problem solving and cooperation
3. Problem-solving research from the Nordic Countries
4. Problem solving from 1995 and onwards
5. Contributions of Malcolm Swan

As you know, Michael Swan will contribute to the course during week 7. We will therefore focus on some of his research literature. It is expected that you read the article published in *Nordic Studies in Mathematics Education* (Swan, 1998) as a preparation for this course week.
As far as problem-solving research is concerned, it is quite probable that you have different backgrounds and experiences from this field. From the research literature we will therefore focus on some main references from each of the categories listed above. You are expected to read thoroughly two articles from each of those categories and read quite a few articles more briefly.

1. Problem solving in a historical perspective

Dewey, J. (1933). *How we think*. Boston: Heath and Company.

Polya, G. (1954). *Mathematics and Plausible Reaso*ning, Vol. I and II. Princeton, NJ: Princeton University Press.

Polya, G. (1957). *How to solve it*. Princeton, NJ: Princeton University Press. (Original work published 1945).

Kilpatrick, J. (1969). Problem solving and creative behavior in mathematics. In J. W. Wilson & L. R. Carey (Eds.), *Review of recent research in mathematics education*. Studies in mathematics series, Vol. *19*, 153-187. Stanford, CA: School Mathematics Study Group.

Lakatos, I. (1976). *Proofs and refutations: The logic of mathematical discovery*. Cambridge: Cambridge University Press.

Lester, F. K. (1980). Research in problem solving. In R. J. Shumway (Ed.), *Research in mathematics education*. Virginia: NCTM.

Mason, J., Burton, L., & Stacey, K. (1982). *Thinking mathematically.* London: Addison-Wesley.

Garofalo, J. & Lester, F. K. (1985). Metacognition and mathematical performance. *Journal for Research in Mathematics Education, 16*, 163-176.

Kilpatrick, J. (1985). Reflection and recursion. *Educational Studies in Mathematics*, *16*, 205 – 214.

Schoenfeld, A. H. (1985). *Mathematical problem solving*. Orlando, FL: Academic Press.

Schoenfeld, A. H. (1987). What's all the fuss about metacognition? In A. H. Schoenfeld (Ed.), *Cognitive Science and Mathematics Education.* Hillsdale NJ: Erlbaum.

Collins, A., Brown, J. S., & Newman, S. E. (1989). Cognitive apprenticeship: Teaching the crafts of reading, writing, and mathematics. In L. B. Resnick (Ed.), *Knowing, learning and instruction: Essays in honor of Robert Glaser*, 453-494. Hillsdale, NJ: Lawrence Erlbaum.

Lave, J., Smith, S. & Butler, M. (1989). Problem solving as everyday experience. In R. I. Charles & E. A. Silver (Eds.), *The teaching and assessing of mathematical problem solving*. Reston, VA: NCTM.

Lester, F. K., Garofalo, J., & Kroll, D. L. (1989). Self-confidence, interest, beliefs, and metacognition: Key influences on problem-solving behaviour. In D. B. McLeod & V. M. Adams (Eds.), *Affect and mathematical problem solving*, 75-88. New York: Springer Verlag.

McLeod, D. B. & Adams, V. M. (1989). *Affect and mathematical problem solving*, New York, Springer Verlag.

Stanic, G. M. A & Kilpatrick, J. (1989). Historical perspectives on problem solving in mathematics curriculum. In R. I. Charles and E. A. Silver (Eds.), *The teaching and assessing of mathematical problem solving*. Reston, VA: NCTM.
Lampert, M. (1990). When the problem is not the question, and the solution is not the answer: Mathematical knowing and teaching. *American Educational Research Journal. 27*, 29-63.
Mason, J. & Davis, J (1991). *Fostering and sustaining mathematics thinking through problem solving*. Victoria: Deakin University Press.
Artzt, A. F. & Armour-Thomas, E. (1992). Development of a cognitive-metacognitive framework for protocol analysis of mathematical problem solving in small groups. *Cognition and Instruction*, *9*, 137-175.
Mason, J. (1992). Researching Problem Solving From the Inside. In J. Ponte, J. Matos & D. Fernandes (Ed.). *Mathematical Problem Solving and New Information Technology: research in Contexts of Practice.* Nato ASI Series F #89, Springer Verlag, London, 17-36.
Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition and sense making in mathematics. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning*, 334-370. New York, Macmillan.
Brown, S. I. & Walter, M. I. (Eds.), (1993). *Problem posing: Reflections and applications*. NJ: Hillsdale, Lawrence Erlbaum.
Lambdin, D. V. (1993). Monitoring moves and roles in cooperative mathematical problem solving. *Focus on Learning Problems in Mathematics*, *15*, 48-64.
Sowder, L. (1993). The looking-back step in problem solving. In S. I. Brown & M. I. Walter (Eds.), *Problem posing: Reflections and applications*, 235-239. NJ: Hillsdale, Lawrence Erlbaum.
Lester, F. K. (1994). Musings about mathematical problem-solving research 1970-1994.*Journal for Research in Mathematics Education*, *25*, 660-675.
Carlson, M. P. & Bloom, I. (2005). (see 4. category)

Main references from 1. category:

Polya, G. (1957). *How to solve it*. Princeton, NJ: Princeton University Press. (Original work published 1945).
Stanic, G. M. A & Kilpatrick, J. (1989). Historical perspectives on problem solving in mathematics curriculum. In R. I. Charles and E. A. Silver (Eds.), *The teaching and assessing of mathematical problem solving*. Reston, VA: NCTM.
Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition and sense making in mathematics. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning*, 334-370. New York, Macmillan.
Lambdin, D. V. (1993). Monitoring moves and roles in cooperative mathematical problem solving. *Focus on Learning Problems in Mathematics*, *15*, 48-64.
Lester, F. K. (1994). Musings about mathematical problem-solving research 1970-1994.*Journal for Research in Mathematics Education*, *25*, 660-675.

2. Problem solving and cooperation

Lewin, K. (1935). *A dynamic theory of personality*. New York: McGraw-Hill.

Deutsch, M. (1949). A theory of cooperation and competition. *Human Relations*, *2*, 129-152.

Damon, W. & Phelps, E. (1989). *Peer interaction, problem solving, and cognition: Multidisciplinary perspectives.* International Journal of Educational Research*,* 13*,* 9-19.

Forman, E. (1989). The role of peer interaction in the social construction of mathematical knowledge. *International Journal of Educational Research*, *13*, 55-70.

Webb, N. M. (1989). Peer interaction and learning in small groups. *International Journal of Educational Research*, *13*, 21-39.

Davidson, N. (1990). Small-group cooperative learning in mathematics. In T. J. Cooney, &C. J. Hirsch (Eds.), *Teaching and learning mathematics in the 1990s*, *1990 Yearbook*. Reston, VA: NCTM.

Johnson, D. W. & Johnson, R. T. (1990). Using cooperative learning in mathematics. In N. Davidson (Ed.), *Cooperative learning in math. A handbook for teachers*, 103-125. Menlo Park, CA: Addison Wesley.

Slavin, R. E. (1990). *Cooperative learning: Theory, research, and practice.* Englewood Cliffs, NJ: Prentice Hall.

Pirie, S. (1991). Peer discussion in the context of mathematical problem solving. In K. Durkin and B. Shire (Eds.), *Language in mathematical education,* 143-161. Milton Keynes: Open University Press.

Webb, N. M. (1991). Task-related verbal interaction and mathematics learning in small groups. *Journal for Research in Mathematics Education*, *22*, 366 - 389.

Cohen, E. G. (1994). Restructuring the classroom: conditions for productive small groups. *Review of Educational Research*, *64*, 1-35.

Cobb, P. (1995). Mathematical learning and small-group interaction: Four case studies. In P. Cobb and H. Bauersfeld (Eds.), *The emergence of mathematical meaning: Interaction in classroom cultures*, 25-129. Hillsdale, NJ: Lawrence Erlbaum.

Qin, Z.; Johnson, D. W. & Johnson, R. T. (1995). Cooperative versus competitive efforts and problem solving. *Review of Educational Research*, *65*, 129 - 143.

Sfard, A.; Nesher, P.; Streefland, L.; Cobb, P. & Mason, J. (1998). Learning mathematics through conversation: Is it as good as they say? *For the Learning of Mathematics 18*, 1, 41-51.

Antil, L. R., Jenkins, J. R., & Wayne, S. K. (1998). Cooperative learning: Prevalence, conceptualizations, and the relation between research and practice. *American Educational Research Journal, 35*, 419 - 454.

Jacob, E. (1999). *Cooperative learning in context. An educational innovation in everyday classrooms*. New York: State University of New York Press.

Springer, L., Stanne, M. E., & Donovan, S. S. (1999). Effects of small-group learning on undergraduate in science, mathematics, engineering, and technology: A meta-analysis. *Review of Educational Research*, *69*, 1, 21-51.

Brodie, K. (2000). Teacher intervention in small-group work. *For the Learning of Mathematics*, *20*, 1, 9-16.

Kramarski, B. & Mevarech, Z. R. (2003). Enhancing mathematical reasoning in the classroom: The effect of cooperative learning and metacognitive training. *American Educational Research Journal*, *40*, 1, 281-310.

Main references from 2. category:

Johnson, D. W. & Johnson, R. T. (1990). Using cooperative learning in mathematics. In N. Davidson (Ed.), *Cooperative learning in math. A handbook for teachers*, 103- 125. Menlo Park, CA: Addison Wesley.

Webb, N. M. (1991). Task-related verbal interaction and mathematics learning in small groups. *Journal for Research in Mathematics Education*, *22*, 366 - 389.

Sfard, A.; Nesher, P.; Streefland, L.; Cobb, P. & Mason, J. (1998). Learning mathematics through conversation: Is it as good as they say? *For the Learning of Mathematics 18*, 1, 41-51.

Springer, L., Stanne, M. E., & Donovan, S. S. (1999). Effects of small-group learning on undergraduate in science, mathematics, engineering, and technology: A meta-analysis. *Review of Educational Research*, *69*, 1, 21-51.

Kramarski, B. & Mevarech, Z. R. (2003). Enhancing mathematical reasoning in the classroom: The effect of cooperative learning and metacognitive training. *American Educational Research Journal*, *40*, 1, 281-310.

3. Problem-solving research from the Nordic Countries

Emanuelsson, G. et al. (red) (1991). *Problemlösning*. Studentlitteratur, Lund.

Borgersen, H. E. (1994). Open ended problem solving in geometry. *Nordisk Matematikkdidaktikk*, *2*, 2, 6-35.

Alseth, B. (1995). Undervisning i problemløsningsstrategier. *Nordisk Matematikkdidaktikk*, *3*, 3, 7-26.

Pehkonen, E. & Törner, G. (1996). Mathematical beliefs and different aspects of their meaning. ZDM, *4***,** 101-108.

Wyndhamn, J. & Säljö, R. (1997). Word problems and mathematical reasoning - a study of children's mastery of reference and meaning in textual realities. *Learning and Instruction*, *7*, 361-382.

Wyndhamn, J. (1998). Problem solving as a metaphor and a practice. Assumptions and conceptions of problem solving and their consequences for activities in the mathematical classroom. In T. Breiteig & G. Brekke, *Proceedings of Norma 98, the Second Nordic Conference on Mathematics Education*, Kristiansand, Norway, 45-58.

Lithner, J. (2000). Mathematical reasoning in task solving. *Educational Studies in Mathematics*, 41, 165-190.

Möllehed, E. (2001). *Problemlösning i matematik. En studie av påverknadsfaktorer i årskurserna 4-9*. Publisert doktoravhandling. Institusjonen för pedagogik. Lärarhögskolan i Malmö.

Bjuland, R. (2002). Problem solving in geometry. Reasoning processes of student teachers working in small groups: A dialogical approach. Published doctoral dissertation. Bergen: University of Bergen.

Grevholm, B. (red.) (2003). *Matematikk for Skolen.* Bergen: Fagbokforlaget.

Lithner, J. (2003). Students' mathematical reasoning in university textbook exercises. *Educational Studies in Mathematics, 52,* 29-55.

Bergsten, C. & Grevholm, B. (Eds.) (2004). *Mathematics and Language*. The 4th Swedish Mathematics Education Research Seminar in Malmö, January 21-22, 2004.
Bjuland, R. (2004). Student teachers' reflections on their learning process through collaborative problem solving in geometry. *Educational Studies in Mathematics*, *55*, 199-225.
Borgersen, H. E. (2004). Open ended problem solving in geometry revisited. *Nordisk Matematikkdidaktikk, 9*, 3, 35-65.
Dahl, B. (2004). Analysing cognitive learning processes through group interviews of successful high school pupils: Development and use of a model. Educational Studies in Mathematics, 56, 129-155.
Ryve, A. (2004). Can collaborative concept mapping create mathematically productive discourses? *Educational Studies in Mathematics*, *56*, 157-177.
Carlsen, M. (in press?) Conceptual understanding of the dot product in a small group of high-achieving students. Collaborative learning in mathematics at upper secondary school.

Suggested references from 3. category:

Borgersen, H. E. (1994). Open ended problem solving in geometry. *Nordisk Matematikkdidaktikk*, *2*, 2, 6-35.
Wyndhamn, J. & Säljö, R. (1997). Word problems and mathematical reasoning - a study of children's mastery of reference and meaning in textual realities. *Learning and Instruction*, *7*, 361-382.
Bjuland, R. (2004). Student teachers' reflections on their learning process through collaborative problem solving in geometry. *Educational Studies in Mathematics*, *55*, 199-225.
Ryve, A. (2004). Can collaborative concept mapping create mathematically productive discourses? *Educational Studies in Mathematics*, *56*, 157-177.
Carlsen, M. (in press?) Conceptual understanding of the dot product in a small group of high-achieving students. Collaborative learning in mathematics at upper secondary school.

4. Problem solving from 1995 and onwards

Mason (1995). Less may be more on a Screen. In L. Burton & B. Jaworski (Eds.). *Technology in mathematics teaching*. *A bridge between teaching and learning*. Chartwell – Bratt.
Goos, M. & Galbraith, P. (1996). Do it this way! Metacognitive strategies in collaborative mathematical problem solving. *Educational Studies in Mathematics*, *30*, 229-260.
Cobb, P., Boufi, A., McClain, K. & Whitenack, J. (1997). Reflective discourse and collective reflection. *Journal for Research in Mathematics Education*, *28*, 258-277.
Kieran, C. & Dreyfus, T. (1998). Collaborative versus individual problem solving: Enteringanother's universe of thought. In A. Olivier & K. Newstead (Eds.). *Proceedings of 22nd PME Conference,* Stellenbosch, South Africa, Vol 3, 112-119.
Mason, J. (1998). Researching From the Inside in Mathematics Education. In A. Sierpinska & J. Kilpatrick (Eds.), *Mathematics Education as a Research Domain: a search for identity*, Kluwer, Dordrecht, 2, 357-378.

Artzt, A. F. & Femia, S. Y. (1999). Mathematical reasoning during small-group problem solving. In L. V. Stiff and F. R. Curcio (Eds.), *Developing mathematical reasoning in grades K-12, National Council of Teachers of Mathematics, 1999 Yearbook*, 115-126.
Reston, VA: NCTM.
Cobb, P. & Bowers, J. (1999). Cognitive and situated learning perspectives in theory and practice. *Educational Researcher*, *28*, 2, 4-15.
DeBellis, V. A. & Goldin, G. A. (1999). Aspects of affect: Mathematical intimacy, mathematical integrity. In O. Zaslavsky (Ed.), *Proceedings of the 23rd PME Conference,* Haifa, Israel, Vol. *2*, 249-256.
Mariotti, M. A. & Maracci, M. (1999). Conjecturing and proving in problem-solving situations. In O. Zaslavsky (Ed.), *Proceedings of the 23rd PME Conference*, Haifa, Israel. Vol. *3*, 265-272.
Mason, J. & Spence, M. (1999). Beyond mere knowledge of mathematics: The importance of knowing-to act in the moment. *Educational Studies in Mathematics*, *38*, 135-161.
Toom, A. (1999). Communications, word problems: Applications or mental manipulatives. *For the Learning of Mathematics*, *19*, 1, pp??
Wood, T. (1999). Creating a context for argument in mathematics class. *Journal for Research in Mathematics Education*, *30*, 171-191.
De Corte, E. , Verschaffel, L. & Opt Eynde, P. (2000). Self-regulation: A characteristic and a goal of mathematics learning. In M. Boekaerts, P. R. Pintrich, & M. Zeidner (Eds.), Handbook of Self-Regulation. San Diego: Academic Press.
Mason, J. (2000). Asking mathematical questions mathematically. *International Journal of Mathematical Education in Science and Technology*, *31*, 97-111.
Verschaffel, L. , Green, B. & De Corte, E. (2000). Making sense of word problem. The Netherlands: Swetz & Zeitlinger.
Brown, S. I. (2001). *Reconstructing School Mathematics: Problems with Problems and the Real World*. Peter Lang Publishing.
Forman, E. & Ansell, E. (2001). The Multiple Voices of a Mathematics Classroom Community. *Educational Studies in Mathematics, 46,* 115-142.
Kieran, C., Forman, E., & Sfard, A. (2001). Learning discourse: Sociocultural approaches to research in mathematics education. *Educational Studies in Mathematics, 46,* 1-12.
Kieran, C. (2001). The mathematical discourse of 13-year-old partnered problem solving and its relation to the mathematics that emerges. *Educational Studies in Mathematics, 46,* 187-228.
Mason, J. (2001). Mathematical Teaching Practices At Tertiary level: Working Group Report. In D. Holton (Ed.), *The Teaching and Learning of Mathematics at University Level*: *An ICMI Study*, Kluwer, Dordrecht, 71-86.
Sfard, A. & Kieran, C. (2001). Cognition as communication: Rethinking learning-by-talking through multi-faceted analysis of students' mathematical interactions. *Mind, Culture, and Activity*, *8*, 1, 42-76.
Zack, V. & Graves, B. (2001). Making mathematical meaning through dialogue: ''Once you think of it, The Z minus three seems pretty weird''. *Educational Studies in Mathematics*, *46*, 229-271.
Brandt, B. (2002). Classroom interaction as multi-party-interaction – methodological aspects of argumentation analysis. In J. Novotná (Ed.), *Proceedings of CERME 2* -

The Second Conference of European Society for Research in Mathematics Education, Praha: Univerzita Karlova, 373-381.
Cobb, P. (2002). Reasoning With Tools and Inscriptions. *The Journal of the Learning Sciences, 11,* 187-215.
Goos, M., Galbraith, P., & Renshaw, P. (2002). Socially mediated cognition: Creating collaborative zones of proximal development in small group problem solving. *Educational Studies in Mathematics*, *49*, 193-223.
Mason, J. (2002). *Researching your own practice. The dicipline of noticing.* London: Routledge/Falmer.
Saxe, G. B. (2002). Children's Developing Mathematics in Collective Practices: A Framework for Analysis. *The Journal of the Learning Sciences, 11,* 275-300.
Schliemann, A. D. (2002). Representational Tools and Mathematical Understanding. *The Journal of the Learning Sciences, 11,* 301-317.
Sfard, A. & McClain, K. (2002). Analyzing Tools: Perspectives on the Role of Designed Artifacts in Mathematics Learning. *The Journal of the Learning Sciences, 11,* 153-161.
Sfard, A. (2002). The Interplay of Intimations and Implementations: Generating New Discourse With New Symbolic Tools. *The Journal of the Learning Sciences, 11,* 319-357.
Crespo, S. (2003). Learning to pose mathematical problems: exploring changes in preservice teachers' practices. *Educational Studies in Mathematics*, *52*, 243-270.
Goos, M. (2004). Learning mathematics in a classroom community of inquiry. *Journal for Research in Mathematics Education*, *35*, 285-291.
Clarke, B et al. (Eds.) (2004). *International Perspectives on Learning and Teaching Mathematics.* Göteborg University: National Center for Mathematics.
Carlson, M. P. & Bloom, I. (2005). The cyclic nature of problem solving: An emergent multidimensional problem-solving framework. *Educational Studies in Mathematics*, *58*, 45-75.

Suggested references from 4. category:

Goos, M. & Galbraith, P. (1996). Do it this way! Metacognitive strategies in collaborative mathematical problem solving. *Educational Studies in Mathematics*, *30*, 229-260.
Cobb, P., Boufi, A., McClain, K. & Whitenack, J. (1997). Reflective discourse and collective reflection. *Journal for Research in Mathematics Education*, *28*, 258-277.
Wood, T. (1999). Creating a context for argument in mathematics class. *Journal for Research in Mathematics Education*, *30*, 171-191.
Sfard, A. & Kieran, C. (2001). Cognition as communication: Rethinking learning-by-talking through multi-faceted analysis of students' mathematical interactions. *Mind, Culture, and Activity*, *8*, 1, 42-76.
Zack, V. & Graves, B. (2001). Making mathematical meaning through dialogue: ''Once you think of it, The Z minus three seems pretty weird''. *Educational Studies in Mathematics*, *46*, 229-271.
Goos, M., Galbraith, P., & Renshaw, P. (2002). Socially mediated cognition: Creating collaborative zones of proximal development in small group problem solving. *Educational Studies in Mathematics*, *49*, 193-223.

Carlson, M. P. & Bloom, I. (2005). The cyclic nature of problem solving: An emergent multidimensional problem-solving framework. *Educational Studies in Mathematics*, *58*, 45-75.
Watson, A. & Mason, J. (in press), *Mathematics seen as a Constructive Activity: the use of learner generated examples*, NJ: Hillsdale, Lawrence Erlbaum.

5. Contributions of Swan

Swan, M. (1998). Discussion Activities, Teacher Beliefs and The Learning of Mature, Low Attaining Students. *Nordic Studies in Mathematics Education*, *6* (3-4), 7-23.
Swan, M. (2000a). GCSE mathematics in Further Education: Challenging beliefs and practices. *The Curriculum Journal*, *11*(2), 199-233.
Swan, M. (2001). Dealing with Misconceptions in Mathematics. In P. Gates (Ed.), *Issues in Mathematics Teaching* (pp. 147-165). London: RoutledgeFalmer.
Swan, M. (2003). Making sense of mathematics. In I. Thompson (Ed.), *Enhancing Primary Mathematics Teaching* (pp. 112-124). Maidenhead: Open University Press.
Swan, M. (2005). *Improving Learning in Mathematics: Challenges and Strategies*. Sheffield: Teaching and Learning Division, Department for Education and Skills Standards Unit.

Reading list MA-607

This list is being updated continually throughout the course. If there is something you have found valuable which is not on the list please let me know to add it.

Against each item, I include the following coding:

*	Important basic text.
*B	You might consider buying this one
**	Special text relating to some aspect of the course
***	General wider reading
****	Particular research studies in mathematics education with useful/interesting methodological perspectives

List starts here:

Details of book	Code
Bassey, M. (1999) Case Study Research in Educational Settings. Buckingham: Open University Press	**
Boaler, J. (1997) Experiencing School Mathematics: Teaching styles, sex and setting. Buckingham: Open University Press	****
Bryman, A. (2001) Social Research Methods. Oxford: Oxford University Press.	*B
Carr, W. and Kemmis, S. (1986) Becoming Critical: Education, Knowledge and Action Research. London: Falmer Press	*
Carspecken, P. F. (1996) Critical Ethnography in Educational Research: A Theoretical and Practical Guide. London: Routledge	***
Cestari, M. L. (1997) Communication in mathematics classrooms: a dialogical approach.	

Unpublished doctoral dissertation, University of Oslo.	**
Clough, P. (2002) Narratives and Fictions in Educational Research. Buckingham: Open University Press	***
Cohen, L. And Manion, L. (1980) Research Methods in Education. London: Routledge. (There should be a new edition of this.)	***
Cooper, B. and Dunne, M(2000) Assessing Children's Mathematical Knowledge: Social class, sex and problem solving. Buckingham: Open University Press.	****
Cooper, P. and McIntyre, D. (1996) Effective teaching and learning: teachers' and students' perspectives. London, Open University Press	**
Davis, P. J. And Hersh, R. (1981) The Mathematical Experience. London: Penguin	*B
Davis, R. B., Maher, C. A. and Noddings, N. (1990) Constructivist Views on the Learning and Teaching of Mathematics. Journal for Research in Mathematics Education, Monograph Number 4. Reston, Va: National Council of Teachers of Mathematics.	**B
Donaldson, M. (1978) Children's Minds. London: Fontana. (Good read. Good introduction to and critique of Piaget's stage theory.)	***
Elliott, J. (1991) Action Research for Educational Change. Buckingham: Open University Press	**
Ely, M., Vinz, R., Downing, M. and Anzul, M. (1997) On Writing Qualitative Research. London: Falmer Press	**
Ernest, P. (1991) The Philosophy of Mathematics Education. London: Falmer Press	**
Glaser, B. G. And Strauss, A. L. (1967) The Discovery of Grounded Theory: Strategies for Qualitative Research. New York: Aldine de Gruyter	**
Goodchild, S. (2001) Students' Goals: a case study of activity in a mathematics classroom. Norway: Caspar Forlag	**
Goodchild, S. and English, L. (2002) Researching Mathematics Classrooms: A critical examination of methodology. London: Praeger [Especially Chapter 1]	**
Gorard, S. (2001) Quantitative Methods in Educational Research. London: Continuum	****
Green, S. B. and Salkind, N. J. (2005) Using SPSS for windows and Macintosh: analyzing and understanding data 4th Edition. Upper Saddle River N.J.: Pearson Prentice Hall.	****
Hammersley, M. (1990) Classroom Ethnography. Buckingham: Open University Press	****
Hart, K. (1981) (ed.). Children's Understanding of Mathematics 11-16. London: John Murray	*B
Herbert C. M. H. (1989) Talking of silence: the sexual harassment of schoolgirls. London, Falmer	**
Hollingsworth, S. (1997) (Ed.) International Action Research: A Casebook for Educational Reform. London: Falmer Press	***
Jaworski, B. (1994) Investigating Mathematics Teaching. London: Falmer Press (especially chapters 2 on Constructivism and 4 on Research Methods)	**
Johnson, D. C. (Ed) (1989). Children's Mathematical Frameworks, 8-13: A Study of Classroom Teaching. London: NFER-Nelson	**
Lakatos, I. (1976) Proofs and Refutations. Cambridge: Cambridge University Press.	**
Lampert, M. (2001)Teaching Problems and the Problems of Teaching. New Haven: Yale University Press.	

Lehrer, K. (1990) Theory of Knowledge. London: Routledge	***
McNiff, J., Lomax, P. and Whitehead, J. (1996) You and your Action Research Project. London: Routledge	**
Maddy, P. (1990) Realism in Mathematics. Oxford: Clarendon Press.	
May, T. (1997) Social Research: Issues, Methods and Process. Buckingham: Open University Press	**
May, T. (1996) Situating Social Theory. Buckingham: Open University Press.	**
Miles, M. B. and Huberman, A. M. (1994) Qualitative Data Analysis: An expanded sourcebook. London: Sage	****
Nesher, P. and Kilpatrick, J. (1990) (Eds.) Mathematics and Cognition. Cambridge: Cambridge University Press.	***
Oppenheim, A. N. (1992) Questionnaire Design, Interviewing and Attitude Measurment. London: Pinter	***
Pring, R. (2000) Philosophy of Educational Research. London: Continuum	***
Punch, K. F. (1998) Introduction to Social Research. London: Sage.	***
Silverman,D. (2000) Doing Qualitative Research - A Practical Handbook. London: Saga Publications	***
Stake, R. E. (1995) The Art of Case Study Research. London: Sage	
Steffe, L. P. and Gale, J. (1995) Constructivism in Education. Hillslade, NJ: Lawrence Erlbaum [Especially chapters by van Glasersfeld (1), Gergen, Bauersfeld, Wertsch and Toma, Confrey, Steffe.]	*
Strauss, A and Corbin, J. (1998) Basics of qualitative research. techniques and procedures for developing grounded theory. London, Sage.	*
Tamhane, A. C. and Dunlop D. D. (2000) Statistics and Data Analysis. Upper Saddle River, NJ.: Prentice Hall.	*B
Teppo, A. (1998) (Ed.) Qualitative Research Methods in Mathematics Education. Journal for Research in Mathematics Education, Monograph Number 9. Reston, Va: National Council of Teacher of Mathematics.	***
Tiller, T., Sparkes, A., Kårhus, S. & Dowling Næss, F. (Eds.) (1995) The qualitative challenge: reflections on educational research. Bergen, Caspar Forlag.	**
Walford, J. (2001) Doing Qualitative Educational Research: A personal guide to the research process. London: Continuum	****
Walker, R. (1985) Doing Research: A Handbook for Teachers. London: Routledge. (There is probably a new edition)	**
Wellington, J. (2000) Educational Research: Contemporary Issues and Practical Approaches. London: Continuum.	**
Wood, D. (1988) How Children Think and Learn. Oxford: Basil Blackwell. (Excellent, easy, introduction to Piaget, Bruner and Vygotsky) (There's probably a new edition.)	*B
Yin, R. K. (1994) Case Study Research. Design and Methods. London: Sage.	**

The following course description illustrates that students and supervisors can create a special reading course adjusted to the needs of a specific student and her project.

MA-609: A Reading Course for the History of Mathematics:

The following plan is destined to give doctor students necessary foundations both in factual and methodical respects for writing a doctor thesis in the field of history of mathematics. The most important basis both for the thematic comprehensiveness of the proposed plan and for its partly rather lose connection to educational themes proper is the specificity of the field of history of mathematics as to methods, facts and topics. Neither can students of mathematics, of mathematical education or of the sciences be expected to have acquired the necessary factual and methodical knowledge in the history of mathematics during their previous academic studies. Nor is knowledge of education-related topics or interest in a career in education a prerequisite to do relevant research in the history of mathematics. For doctor students going in the direction of history of mathematics in a more specific sense, the proposed course might serve as an alternative to the regular doctor courses in mathematics education. This does not preclude, however, that doctor students interested in pursuing a career in mathematics education are also eligible to take this course in agreement with their supervisors. Anyhow there are lots of points of common interest, and some literature given below – most notably Davis/Hersh, Descartes, Euclid, Jahnke et.al. and Bekken/Mosvold - is also bearing immediately on topics of mathematical education.

The literature shall give a broad spectrum of different perspectives on the history of mathematics and facilitate access to the main existing literature, with particular emphasis on literature in English. The aspects addressed in particular are bibliographic-methodical (Dauben/Scriba, Sarton), general chronological and content-oriented (Katz), discipline, -problem-, or notion-oriented (Gericke, Lützen, Tietze), general epistemological (Kuhn), sociological-institutional (Mehrtens, Siegmund-Schultze), ethnological-cultural (Joseph) biographical work (Stubhaug), original (Descartes, Euclid, Ewald, Fauvel/Gray), methodological-philosophical-pedagogical-psychological (Davis/Hersh, Jahnke, Bekken/Mosvold).

Dependent on the particular research area of the doctor student (within or without the history of mathematics) the following literature shall be read by the student under differing points of view (usually not in their totality, except for shorter texts such as Sarton) which have to be clarified by a **proper specification given by the supervisor**.

As a general stipulation it is proposed that each of the 19 pieces of literature shall be read by the student under the given specification. About **5-10 pages of notes and commentary**, preferably with own opinions on what was read, shall be delivered **on each item** to the supervisor who returns the notes with commentary.

At the end of the reading course an **oral exam** shall discuss the notes and further questions both of a more general character and of importance for the particular thesis to be written by the doctor student.

Ending this plan for a reading course I give short descriptions of the content of the different pieces of literature. Particular points of reference have to given on a separate sheet in dependence on the student's research area.

Short descriptions (abstracts) of the literature:

Bekken/Mosvold
This is together with Jahnke et al. one of the rare collections which explicitly address the connection between the history of mathematics and education. It has also interesting articles on the history of Scandinavian mathematics.

Dauben/Scriba
This is the standard work of methodical self-reflection by the present generation of leading historians of mathematics. It gives main literature of the history of mathematics, description of methods, commentary on national traditions in the history of mathematics, short biographies both of mathematicians and historians of mathematics.

Davis/Hersh
This is a very successful overview of mathematical research methods in historical perspective with many relevant and richly illustrated examples. The traditional narrow focus on logical structure in mathematics is broadened by considering intuition and sociological factors in the genesis of mathematics.

Descartes
Descartes "Geometry" (1637), given in a bilingual (English-French) edition, is a masterpiece of mathematics (analytic geometry) in its connection with philosophical ideas, not least relevant to mathematics education. The doctor student can catch both the spirit of original research and the method of editing such texts (with rich footnotes).

Euclid
Euclid's "Elements" (about 300 BCE) remain a standard text of reference for the history of mathematics, and each historian of mathematics has to read it at some point of time. The logical structure with impact on modern axiomatics, the pedagogical relevance (Euclid being on of the first famous teachers of mathematics), not least the problems treated in the book continue to be important both for mathematics and its history.

Ewald
This is a collection of sources for the history of mathematics with emphasis on the newer periods and topics from logics and set theory. It gives crucial work by Cantor, Dedekind and others in English translation.

Fauvel/Gray
This is a very useful collection of excerpts from original text of the history of mathematics in English translation and with commentary. The period ranges from pre-Greek (Babylonian, Egyptian) up till 19th century logic and calculation machines.

Gericke
This is one of the few available texts for the history of mathematics in a Scandinavian language (Danish) which can, in addition, be read in the German original. Since the notion of number is central in mathematics a general overview over its development from antiquity to the 19th century is basic and useful for any student of the history of mathematics.

Jahnke
This is together with Bekken/Mosvold one of the rare collections which explicitly address the connection between the history of mathematics and education. It contains also some historical articles in a more narrow sense. Computing practices, proof structures in Chinese mathematics, experiences with class room teaching in the history of mathematics are addressed among other topics

Joseph
This is a standard popular presentation of the history of non-European mathematics, particular Indian, Chinese, and Arabic-Islamic. Reading these sources prevents a one-sided look on European/North-American mathematics and its logical and sociological structure.
Katz
Katz is one of many general histories of mathematics which now exist in the English language. It is particularly strong in giving modern literature and assignments for students. Also the pedagogical point of view is often present. It has to be read critically though because it contains several mathematical errors and unduly shortened mathematical arguments. But pointing the doctor student to these shortcomings allows sharpening her/his critical mind.
Kuhn
The Structure of Scientific Revolutions is a classical text for the philosophy of science, particularly physics. It avoids the restrictions of a purely rationalistic epistemology and takes sociological factors in the genesis and acceptance of scientific theories, in particularly the values of the "scientific community," into account. Although the text reflects barely on the history of mathematics, it is nevertheless very much oriented towards history (with Newton as a main figure, who was also one of the most important mathematicians in history). Parallel attempts to use the notion of revolution in the history of mathematics have been made by other authors such as M.Crowe, H.Mehrtens, and M.Otte (in Jahnke et.al.).
Lützen
On this book applies partly what has been said on Gericke with respect to the language. It is oriented towards the classical (Greek) geometric problems and in this respect it is a good complement to Gericke
Mehrtens et al.
This book is one of the few which address sociological and institutional problems of the history of mathematics in a more systematic way. Main focuses are the French and German models of mathematics and university education in this field.
Sarton
Although nearly 70 years old, this text gives still a good introduction into methods and older literature of the history of mathematics. It seeks at the same time connection to neighbouring fields, Sarton being the pioneer of modern (after WWII) historiography of science.
Siegmund-Schultze
This book provides biographical sources on modern mathematicians and deals with important institutional and political developments the 20^{th} century mathematics, particularly institutes and emigration of mathematicians.
Stubhaug
The books by Stubhaug on Abel and Lie give much cultural background for the history of mathematics in Norway and serve therefore particularly an understanding of the local (Norwegian) traditions of the history of mathematics. Any student of the history of mathematics at Agder University College should read the two books. In the English translation there are unfortunately many mathematical mistakes.
Tietze
This is the English translation of a successful German book on central mathematical problems and their development during history. It gives valuable historical literature and bears the stamp of research, the author being himself an important mathematician.

Literature:

Joseph W. Dauben and Christoph J.Scriba (Hrg.): *Writing the History of Mathematics: Its Historical Development*. Birkhäuser: Basel, Boston, Berlin 2002

Bekken, O.and R.Mosvold (eds.) Study the Masters: The Abel-Fauvel-Conference; Göteborg 2003.

Davis, Ph. and R.Hersh: *The Mathematical Experience*; Boston 1981.

Descartes, R.: *The Geometry of René Descartes, with a facsimile of the first edition*; New York: Dover 1954 (faksimile (1637) + translation and commentary: D.E.Smith and M.L.Latham)

Euclid: *The Thirteen Books of Euclid's Elements, translated from the Text of Heiberg with introduction and commentary by Thomas L.Heath*; New York: Dover 1956.

Ewald, W.: *From Kant to Hilbert: A Source Book in the Foundations of Mathematics*; Oxford 1996 (2 bind)

Fauvel, J. and J.Gray (eds.): *The History of Mathematics. A Reader*; Milton Keynes: The Open University 1987.

Gericke, H.: *Talbegrebets Historie*; Aarhus Universitet 1994, 1996 (2nd edition), translated from the German original of 1970 by Kirsti Andersen and Kate Larsen, Danish

Jahnke, H.N., N.Knoche and M.Otte (eds.): *History of Mathematics and Education: Ideas and Experiences*; Göttingen: Vandenhoeck & Ruprecht 1996.

Joseph, G.G.: *The Crest of the Peacock: Non- European Roots of Mathematics*; London: Penguin 1991.

Katz, V.J.: *A History of Mathematics*; Reading, MA etc.: Addison Wesley 1998.

Kuhn, Th.: *The Structure of Scientific Revolutions*; Chicago 1962 (Dansk København 1973).

Lützen, J.: *Cirklens kvadratur, Vinklens tredeling, Terningens fordobling: Fra oldtidens geometri til moderne algebra*; Herning: Systime 1985 (Dansk)

Mehrtens, H., H.Bos and I.Schneider (eds*.): Social History of Nineteenth Century Mathematics*; Basel: Birkhäuser 1981.

Sarton, G.: The Study of the History of Mathematics; Cambridge, MA 1936. [Reprint New York: Dover 1957]

Siegmund-Schultze, R.: *Rockefeller and the Internationalization of Mathematics Between the Two World Wars. Documents and Studies for the Social History of Mathematics in the 20th century*; Basel etc.: Birkhäuser 2001.

Stubhaug, A.: Niels Henrik Abel and his times; Berlin: Springer 2000 [orig. Oslo 1996]

Stubhaug, A.: *Det var mine tankers djervhet. Matematikeren Sophus Lie*. Oslo: Aschehoug 2000. [English: The mathematician Sophus Lie: it was the audacity of my thinking. Berlin: Springer 2002].

Tietze, H.: *Famous Problems of Mathematics*; Baltimore: Graylock Press 1965

Appendix 3

Views of knowing and learning: Constructivism, socio-cultural theory and the relationships between them, with special emphasis on the writings of Paul Cobb

Course provider: The Danish University of Education in cooperation with the Danish National Graduate School of Research in Science and Mathematics Education (NADIFO), the Nordic Graduate School in Mathematics Education, and Learning Lab Denmark.

Dates: Seminars on September 12-15 and November 30 to December 2. Between the two seminars there shall be continued web-based discussions on the main theme of the course.

Deadline for applications:

Venue: The Danish University of Education.

ECTS-credits:

Teachers:
Associate professor Seth Chaiklin, the Danish University of Education.
Professor Paul Cobb, Vanderbilt University, Nashville, USA.
Associate professor Jeppe Skott, the Danish University of Education.

Limit: 20 students.

Course language: English.

Content:

The main aim of the course is to provide the students with a thorough introduction to developments and current understandings of the relationship between individual and social perspectives on knowing and learning in mathematics and science education. More specifically, the course intends to

(1) familiarise the students with constructivism and socio-cultural theory, two of the most significant theoretical sources of inspiration on the theory of mathematics and science education over the last decades;
(2) develop the students' understandings of how dominant conceptions of the relationship between constructivism and socio-cultural theory have changed since the 1980s;
(3) discuss the writings of Paul Cobb as a body of work in its own right and as an example of the developments mentioned under (2);
(4) develop the students' understanding of how significant developments in the theory mathematics and science education have resulted from the interplay

between external sources of inspiration and empirical studies of classroom interactions.

Programme:

On the first day of each of the two seminars we shall meet from 10.00-12.00 and from 13.00-16.30. On all other days we shall meet from 9.00-12.00 and 13.00-16.30.

The September seminar:
12 September: Socio-cultural theory.
10.00-11.00: Welcome and short presentations.
11.00-12.00 and 13.00-16.30: Presentation and discussion of the literature by Vygotsky and Davidov. (Seth Chaiklin)

13 September: Constructivism
9.00-12.00: Presentation and discussion of the literature by von Glasersfeld (Jeppe Skott).
13.00- 16.30: Presentation and discussion of the literature by Piaget. (Seth Chaiklin).

14 September:
9.00-12.00: Group work: Presentations of the participants projects in small groups around the theme of the role of theories of knowing and learning, incl. a discussion of how the contents of the two first days feeds into their projects. (Seth Chaiklin and Jeppe Skott).
13.00-16.30: The relationship between constructivism and socio-cultural theory in mathematics education. Discussion of the Lerman-Steffe/Thompson controversy and of the perspectives offered by Confrey and Sfard. (Jeppe Skott).

15. September:
9.00-12.00: Introducing the work of Paul Cobb. (Jeppe Skott).
13.00-16.30: Reading and writing research papers. (Jeppe Skott).

Between the seminars
Two sets of activities are to take place between the two seminars. First the students are to use a web-based discussion group to present and discuss problems in relation to three of Cobb's papers.
Second they are to write an 8-10 page paper in part referring to the relationship between individual and social perspectives on learning and knowing. The format of the paper may for instance be (a draft) of a conference paper, a part of a chapter in the thesis or a section in a longer article. This paper must be ready at least two weeks before the second seminar. The papers will then be grouped in fours or fives and the participants will read the papers of the other members in their group before the second seminar.

The November-December seminar
30 November:
10.00-12.00: Group work: Similarities and differences between the papers in the group: What use is made of individual and social perspectives on knowing and learning? (Jeppe Skott).

13.00-16.30: Workshop: The implications of a situated view of learning for the design of research and instruction. (Paul Cobb)

1 December:
9.00-12.00: Workshop ctd.: The implications of a situated view of learning for the design of research and instruction. (Paul Cobb).
13.00-16.30: Group work, ctd.: preparing a presentation of key problems for 2 December. (Paul Cobb, Jeppe Skott).

2 December:
9.00-12.00: Group presentations and discussion. (Paul Cobb and Jeppe Skott).
13.00-16.30: Group presentations and discussion. (Paul Cobb and Jeppe Skott).

Form

During the seminars, a mixture of lectures and group work is used. Combined with the participants' prior reading, the lectures are to facilitate and qualify group discussions on individual and social perspectives on knowing and learning. In turn the group work is to ensure that these perspectives - main theme of the course - feed into and inform the participants' research projects. This latter intention is also to be realised through the activities between the seminars, i.e. through the reading, web-based discussions and writing that to a large extent form the basis of the second seminar.

Preparation demands:

This course is targeted at research students who need a sound foundation in learning theory for their research projects and who have some preliminary expectations as to what role learning theory is to play in their project. For each of the first of the two seminars, the literature below is divided in two. First it consists of literature which the participant is expected to have familiarised him- or herself with in advance. Second there is a recommended list of literature for further reading.
It is vital to the success of the course that the participants complete the paper they are to write two weeks before the second seminar. Therefore certificates for completion of the course will only be issued to those who have met this deadline.

Literature:

For the September seminar:
Cobb, P. (1994). Where is the mind? Constructivist and sociocultural perspectives on mathematical development. *Educational Researcher, Vol. 23(7)*, 13-20.
Davydov, V. V. (1995). The influence of L. S. Vygotsky on education, theory, research, and practice. *Educational Researcher, 24*(3), 12-21.
Glasersfeld, E. von (1995a). *Radical constructivism. A way of knowing and learning*. London: The Falmer Press. Only pp. 53-75 & 89-128.

Glasersfeld, E. von (1995b). A constructivist approach to teaching. In L.P. Steffe & J. Gale (eds.): Constructivism and Education (3-16). Hillsdale, New Jersey: Lawrence Erlbaum.

Glasersfeld, E. von (2000). Problems of Constructivism. In L.P. Steffe & P.W. Thompson (eds.): *Radical constructivism in action. Building on the pioneering work of Ernst von Glasersfeld* (3-10). London: RoutledgeFalmer.
Lerman, S. (1996): Intersubjectivity in mathematics learning: A challenge to the radical constructivist paradigm? *Journal for Research in Mathematics Education, Vol. 27(2)*, 133-150.
Lerman, S. (2000): A case of interpretations of social: A response to Steffe and Thompson. *Journal for Research in Mathematics Education, vol. 31(2),* 210-227.
Sfard, A. (2003): Balancing the unbalanceable: The NCTM standards in light of theories of learning. In J. Kilpatrick, W.G Martin, & D. Schifter: *A Research Companion to Principles and Standards for School Mathematics*. Reston, VA: NCTM, 353-392.
Steffe, L.P. & Thompson, P.W. (2000): Interaction or intersubjectivity? A reply to Lerman. *Journal for Research in Mathematics Education, vol. 31(2),* 191-209.
Vygotsky, L. S. (1987). Thinking and speech (N. Minick, Trans.). In R. W. Rieber & A. S. Carton (Eds.), *The collected works of L. S. Vygotsky: Vol. 1. Problems of general psychology* (pp. 39-285). New York: Plenum Press. (Original work published 1934).
Chapter 6: The development of scientific concepts in childhood (only pp. 167-214)

Recommended literature:

Chaiklin, S. (2003). The zone of proximal development in Vygotsky's theory of learbning and school instruction. In A. Kozulin, V. Ageyev, B. Gindis, & C. Miller (Eds.), *Vygotsky's educational theory in cultural context* (pp. 39-64). Cambridge: Cambridge University Press.

Confrey, J.: How compatible are radical constructivism, sociocultural approaches and social constructivism? In L.P. Steffe & J. Gale (eds.): Constructivism and Education (185-225). Hillsdale, New Jersey: Lawrence Erlbaum.

Galperin, P. I. (1979). The role of orientation in thought. *Soviet Psychology, 18*(2), 84-99.
Kieran, T. (2000): Dichotomies or binoculars: Reflections on the papers by Steffe and Thompson and by Lerman. *Journal for Research in Mathematics Education, Vol. 31(2)*, 228-233.
Skott, J. (2004): The forced autonomy of mathematics teachers. *Educational Studies in Mathematics, 55,* 227-257.
Vygotsky, L. S. (1997). *The collected works of L. S. Vygotsky: Vol. 4. The history of the development of higher mental functions* (M. Hall, Trans.; R. W. Rieber, Ed.). New York: Plenum Press. (Original work written 1931)
Chapter 1 The problem of the development of higher mental functions(pp. 1-26)
Chapter 2 Research method (only pp. 59-63)
Chapter 5 Genesis of higher mental functions (at least bottom from p. 103 through 110)

For the November-December seminar:

Cobb, P. & Steffe, L.P.(1983): The constructivist researcher as teacher and model builder. *Journal for Research in Mathematics Education, Vol. 14(2)*, 83-94.
Cobb, P. (1989): Experiential, cognitive and anthropological perspectives in mathematics education. *For the Learning of Mathematics, 9(2)*, 32-42.

Cobb, P. (2000). The importance of a situated view of learning to the design of research and instruction. In J. Boaler (Ed.), *Multiple perspectives on mathematics teaching and learning* (pp. 45-82). Stamford, CT: Ablex.
Cobb, P. (2001). Supporting the Improvement of learning and teaching in social and institutional context. In S. Carver. & D. Klahr (Eds.), *Cognition and Instruction: Twenty-Five Years of Progress* (pp. 455-478). Mahwah, NJ: Lawrence Erlbaum Associates.
Cobb, P. (2002). Reasoning with tools and inscriptions. *Journal of the Learning Sciences, 11*, 187-216.
Cobb, P., Boufi, A., McClain, K., & Whitenack, J. (1997): Reflective discourse and collective reflection. *Journal for Research in Mathematics Education, Vol. 28(3)*, 258-277.
Cobb, P., Gravemeijer, K., Yackel, E., McClain, K., & Whitenack, J. (1997): Mathematizing and symbolizing: The emergence of chains of significance in one first-grade classroom. In D. Kirshner & J.A. Whitson: *Situated cognition. Social, semiotic, and psychological perspectives,* 151-235
Gravemeijer, K., Cobb , P., Bowers, J., & Whitenack (2000). Symbolizing modelling and instructional design. In p. Cobb, E. Yackel, & K. McClain: *Symbolizing and communicating in mathematics classrooms. Perspectives on discourse, tools, and instructional design*. Mahwah, New Jersey: Lawrence Earlbaum Associates, 225-273.

Responsible for the course:
Jeppe Skott, e-mail: skott@dpu.dk

Appendix 4

MATHEMATICS AND GENDER

This is a unique and exciting opportunity to attend a specialist course given by a leading international researcher and expert. The atmosphere will be informal and student-centred and the discussion and sharing of students' own ideas and experiences will be a central theme of the course.

DESCRIPTION OF DOCTORAL LEVEL COURSE

TITLE: Mathematics and Gender

LECTURER: Paul Ernest

COURSE DIRECTOR: Ingvill Merete Stedøy

CREDIT VALUE: 5 ECTS

DURATION OF COURSE: Two periods of 2,5 days, c. 25 hrs. contact time.

DATES: 5, 6, 7 April for first part and 10, 11, 12 May 2006 for second part (finishing lunch-time on Fridays).

LOCATION: Norwegian Center for Mathematics Education, NTNU, Trondheim

AIMS

The aims of the course are:

1. to review some of theories and literature on philosophy of mathematics and gender and mathematics,
2. to explore current debates and controversies in this field, focusing mainly on gender,
3. to explore the variety of explanations and mechanisms proposed to account for real or perceived gender differences in achievement and participation rates in mathematics,
4. to consider briefly broader issues concerning mathematics and equal opportunities, including multicultural mathematics.

ASSESSMENT METHOD: 5000 word essay (for those students who want the credits)

TEACHING METHODS: Lectures, discussions, workshops, video presentations, student presentations, written course handbook.

The course content will be covered in an set of extensive lecture notes (the course handbook) available before the first meeting. The lecturer will provide overview lectures of the main ideas in seminar form, and students will be encouraged to discuss the issues and relate them to their own experiences. The whole atmosphere will be informal and student-

centred. Where relevant video clips and practical workshops will be used to illustrate and work with the ideas. Students will be required to choose a topic related to a theme of the course to investigate. During the second part of the course students will make short informal presentations to the group of the questions they have chosen to investigate, and the ideas behind them, in order to get constructive feedback and suggestions from the group. Those who wish to will then develop their topics for the 5000 word essay for assessment. This can be submitted in English or an approved Scandinavian language.

INDICATIVE CONTENT

The nature of mathematics, public images of mathematics, overview of recent work in the philosophy of mathematics.

Mathematics and values. The models of Perry, Gilligan, and Belenky *et al.* applied to mathematics and gender.

British and international data on gender differences in achievement and participation rates in mathematics (and science).

Different ways of conceptualising the 'gender and mathematics problem'. Different ideological perspectives

Biological, psychological and social theories of difference.

Theorising the social construction of differences. Theories of social influence.

Broader aspects of equal opportunities and social justice.

Values, mathematics and the curriculum. Multicultural mathematics.

INDICATIVE READING LIST

Atweh, B. and Cooper, T. (1995) The construction of gender, social class and mathematics in the classroom, *Educational Studies in Mathematics* Vol. 23, No. 3, 293-310.

Belenky M F, Clinchy B M, Goldberger N R and Tarule J M (1986) *Women's Ways of Knowing*, New York: Basic Books

Benbow, C. P. and Lubinski, D. (1993) Psychological profiles of the mathematically talented: some sex differences and evidence supporting their biological basis. In G. R. Bock and K. Ackrill, Eds, *Ciba Foundation Symposium 178: the origins and development of high ability*, New York: Wiley.

Bergsten, C. and Grevholm, B. Eds. (2003) *Challenges in mathematics education, Proceedings of MADIF2*, Linköping: SMDFs skriftserie.

Bergsten, C. and Grevholm, B. Eds. (2003) *Mathematics and language, Proceedings of MADIF2*, Linköping: SMDFs skriftserie.

Bergsten, C. and Grevholm, B. Eds. (2005). *Conceptions of mathematics, Proceedings of Norma01*, Third Nordic Conference on Mathematics education, Linkøping: SMDFs skriftserie.

Bergsten, C., Dahland, G. and Grevholm, B. Eds. (2002) *Research and action in the mathematics classroom*, Linköping: SMDFs skriftserie.

Biehler R, Scholz R W, Straesser R and Winkelmann (1984) (Eds) *The Didactics of Mathematics as a Scientific Discipline*, Dordrecht: Kluwer

Bishop, A. Ed. (1996) *International Handbook of Research in Mathematics Education*, Dordrecht, Holland: Kluwer.

Brandon, P. R. and Jordan, C. (1994) Gender Differences favoring Hawai'i girls in mathematics achievement: Recent findings and hypotheses, *Zentralblatt fur Didaktik der Mathematik*, 94(1) 18-21.

Burton, L. Ed (1986) *Girls into Maths Can Go*, London, Cassell.

Burton, L. Ed. (1990) *Gender and Mathematics: An International Perspective*, London: Cassell

Davis, P. J. and Hersh, R. (1980) *The Mathematical Experience*, Boston: Birkhauser.

Dunne M and Johnston J (1992) An Awareness of Epistemological Assumptions: The Case of Gender Studies, *International Journal of Science Education*, 14 (5) 515-526

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BRIEF DESCRIPTION OF PAUL ERNEST

Paul Ernest is Professor of the Philosophy of Mathematics Education at Exeter University, UK. He directs the specialist masters and doctoral programmes in mathematics education at Exeter which continue to attract students from almost every continent. He is well known internationally for his research and conference contributions and has published over 200 papers, chapters and books across the field of mathematics education. His main research interests concern fundamental questions about the nature of mathematics and how it relates to teaching, learning and society. He is best known for his work on philosophical aspects of mathematics education and he edits the international web-based *Philosophy of Mathematics Education Journal*, located at <http://www.ex.ac.uk/~PErnest/>. His books include *The Philosophy of Mathematics Education*, Falmer 1991, and *Social Constructivism as a Philosophy of Mathematics,* SUNY Press, 1998. He edits the Routledge/Falmer Press series Studies in Mathematics Education. Recently he has been working on the semiotics of mathematics education and globalization.

Paul was born in the USA to an American father of Russian Jewish origin and Swedish mother. He lived in Sweden for 3 years as a child before settling in the UK. His family connections and more recently his association with NCM in Göteborg have brought him to Sweden more than 20 times, and he speaks Swedish. He has visited Norway and Denmark many times, as well as single visits to Iceland and Finland.

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Appendix 5

4F

Graduate course for doctoral students in didactics of mathematics:

Theory and practice of four French frameworks for research in the didactics of mathematics

Organised by the Centre for Science Education, University of Copenhagen and the Nordic Graduate School in Mathematics Education.

This course aims to introduce participants to four related theoretical frameworks for research in the didactics of mathematics, to foster reflection and discussion among participants on the nature of the discipline in the light of these theories, and to enrich the participants' own work by relating it to one or more of these frameworks. The four frameworks are:

- The theory of conceptual fields (due mainly to Gérard Vergnaud)
- The semiotic approach (due mainly to Raymond Duval)
- The theory of didactical situations in mathematics (due mainly to Guy Brousseau)
- The anthropological approach (due mainly to Yves Chevallard)

We shall read both "basic theory" and related, more specialised research articles. All of the required readings will be in English, but some additional (and optional) texts may be in French. The course language will be English or, if all participants indicate this preference *at the time of registration*, Danish/Norwegian/Swedish.

Scientific organiser: Professor Carl Winsløw, Centre for Science Education, University of Copenhagen

Timeline of the course.

0. November 2004: The first texts are sent to the participants, along with questions to reflect on while reading.

1. January 6-7, 2005: First session, with the following agenda:
   - Short introductions by participants of their projects and interests
   - Presentation and discussion of the four frameworks based on the first texts.
   - A new package of texts (research articles based on the frameworks) will be distributed at the end of this session. These will, as far as possible, be related to the projects and interests of participants.

2. February 3-4, 2005: Second session, treating the texts of the second package and their relations to the projects of the participants. Participants are expected to prepare a short oral presentation of their ideas regarding their own project, which will be discussed among the whole group.

3. March 10-11, 2005: Third and final session, a "mini-conference" where each participant will present a paper on aspects of their own projects which can be usefully related to one or more of the four frameworks (this paper must be finished and distributed via email to all participants before the session according to a deadline fixed at the second session).

Venue: All sessions will take place at the Centre of Science Education, University of Copenhagen. Travel and lodging must be organised by participants. (Regarding a possibility for travel support, see below)

Prerequisites: Participants are expected to be doctoral students in the didactics of mathematics with some general acquaintance of the field as such, and a clearly formulated research project which could be in any stage between beginnings and almost finished. We shall also assume some familiarity with cognitive psychology (of Jean Piaget) and its use in educational theory. A good and sufficient preparation for this latter area can be found in Chap. 8 of Svein Sjøberg: Naturfag som allmenndannelse – en kritisk fagdidaktikk (ad Notam Gyldendal, Oslo, 1998 and later).

Workloads and credits: The course will require about 200 hours of work, corresponding to a course credit of **7.5 ECTS points** (pre-approbation of course credits must be obtained from home institution). The work includes readings, oral presentations and final essay (of about 10 pages). Course assessment will be based on the final essay. A statement of satisfactory completion will be available.

Number of participants: 10 (both max. and min.)

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Common literature for the first session:

1. Yves Chevallard: *Didactique? you must be joking! A critical comment on terminology.* Instructional Science 27 (1999), 5-7.
2. Guy Brousseau: *Research in Mathematics Education: Observation and ... mathematics.* In: Proceedings of CERME 1, European research in mathematics education. Vol. 1. Proceedings. Editor(s): Schwank, Inge Forschungsinstitut fuer Mathematikdidaktik e.V., Osnabrueck (Germany) 1999. p. 35-49. Forschungsinst. fuer Mathematikdidaktik, Osnabrueck.
3. Guy Brousseau: *Theory of Didactical Situations in mathematics*, Dordrecht, Kluwer, 1997, pp. 3-75.
4. Barbé, J., Bosch, M., Espinoza, L. et Gascón, J. (2005): Didactic restrictions on teachers practice - the case of limits of functions at spanish high schools. *Educational Studies in Mathematics* **59** (1-3), 235-268.
5. Gérard Vergnaud: *The theory of conceptual fields.* In: L. P. Steffe, P. Nesher, P. Cobb, G. Goldin, B. Greer (eds), Theories of Mathematical Learning. Lawrence Erlbaum,1996, pp. 219-240.
6. Raymond Duval: *The cognitive analysis of problems of comprehension in the learning of mathematics. Mediterranean Journal for Research in Mathematics Education* 1(2), 1-16.

Litterature for the second session (choises possible):

1. Michèle Artigue: *Didactical engineering as a framework for the conception of teaching products*. In: R. Biehler et al., *Didactics of mathematics as a scientific* discipline, 27-39. Dordrecht: Kluwer, 1994.

2. Michèle Artigue: *Learning mathematics in a CAS environment: the genesis of a reflection about instrumentation and the dialectics between technical and conceptual work*. Int. J. of Computers for Math. Learn. 7 (2002), 245-274. (Appendix: comment, pp. 293-299, same J.)
3. Raymond Duval: *L'apprentissage de l'algèbre et le problème cognitif de la designation des objets.* Actes du Sém. Franco-Italien sur l'algèbre (in press)
4. Michèle Artigue et al. : *Teaching and learning algebra: approaching complexity through complementary perspectives.* In: H. Chick et al. (eds), *Proceedings of the 12th ICMI Study Conference The Future of the Teaching and Learning of Algebra,* vol.1, 21-32, University of Melbourne.
5. Régine Douady: *Relation function/al algebra: an example in high school* (age 15-16). In: I. Schwank (ed), *European research in mathematics education*. Vol. 1, pp. 113-124. Osnabrück: Forschungsinstitut fuer Mathematikdidaktik.
6. Michèle Artigue: *The teaching and learning of mathematics at the university level.* Notices of the AMS, Dec. 1999, 1377-1385.
7. Carl Winsløw: *Research and development of university level teaching: The interaction of didactical and mathematical organizations.* Paper accepted for CERME-5, 2005.
8. Virginia Warfield: *Calculus by scientific debate.* Unpubl. Manus.
9. Isabelle Bloch: *From academic mathematics to mathematics to be taught: situations for mathematics teachers' education.* Preprint 2004
10. Alain Mercier et. al.: *How social interactions within a class depend on the teachers' assessment of the pupils' various mathematical capabilities: a case study.* In: I. Schwank (ed), *European research in mathematics education*. Vol. 1, pp. 342-353. Osnabrück: Forschungsinstitut fuer Mathematikdidaktik.
11. Nicholas Balacheff: *A modeling challenge: untangling learners' knowing.* Paper presented at *Journées internationales d'Orsay sur les sciences cognitives* (2000).
12. Jean-Philippe Drouhard et al.: *Necessary mathematical statements and aspects of knowledge in the classroom.* In: I. Schwank (ed), *European research in mathematics education*. Vol. 1, pp. 320-330. Osnabrück: Forschungsinstitut fuer Mathematikdidaktik.
13. Luis Radford: *On heroes and the collapse of narratives: a contribution to the study of symbolic thinking.* In A. Cockburn et al., *Proc. of PME 26*, vol. 4, 81-88. Norwich: UEA.
14. Heinz Steinbring: *What makes a sign a mathematical sign? An epistemological perspective on mathematical interaction.* Ed. Stud. in Math., to appear.