

## **Nordic doctoral programmes in didactics of mathematics**

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### **Introduction**

#### *A short historical background of doctoral programmes in didactics of mathematics*

In Sweden the academic subject pedagogy (or general education) developed in the beginning of the 20<sup>th</sup> century and inside this field it was possible to carry out doctoral studies concerning mathematics education. One of the first doctoral dissertations about mathematics learning was the one by K. G. Jonsson in 1919, which is an interview study with pupils: Investigations concerning the conditions and course of problem calculations (*Undersökningar rörande problemlösningens förutsättningar och förlopp*, my translation) (Bergsten, 2002). According to Bergsten (2002) 23 Swedish Ph D works during the period 1919-1999 could be classified as belonging to the field of mathematics education. It was not until the mid 1990 that separate doctoral programmes in mathematics education were constructed. This happened around the same time in the universities in Luleå, Umeå and Gothenburg. The first dissertation was presented in Luleå University of Technology by Andrejs Dunkels (1996), and it was followed by Tomas Bergqvist in Umeå University (2001). In 2000 the Swedish Bank Centennial Foundation decided to offer 45 million Swedish crowns for a National Graduate School in mathematics with didactical direction. This led to the development of new doctoral programmes in seven more departments (Leder, Brandell & Grevholm, 2004). More about the graduate school will be presented below. The development in the other Nordic countries was similar to this example from Sweden, which will be visible from later sections in the paper.

#### *About terminology*

In Sweden and the Nordic countries mathematics education is called ‘matematikdidaktik’ (or similar words in the different languages), didactics of mathematics, thereby following the German and French tradition rather than the Anglo-Saxon, when it comes to the notion. In Sweden mathematics education is translated to ‘matematikutbildning’, which means education in mathematics, including school level and other levels. Thus there is a risk of misinterpretations when using the word mathematics education as a name for the research field. Here I will use both these notions interchangeably.

#### *My background and experiences of doctoral programmes in mathematics education*

Since 2004 I am the director of the Nordic Graduate School in Mathematics Education, situated at the University of Agder (UiA). During 2000-2006 I was the vice chair of the Swedish Graduate School in Mathematics with Didactical Direction, which included 10 institutions in Sweden and had 21 doctoral students. I will say more about that later. I was appointed full professor in didactics of mathematics at the University of Agder in 2002 and before that I worked as professor of mathematics and learning at Luleå University of Technology during 2001-2002. Still earlier I have worked at the mathematics department at Lund University for 10 years, as mathematics teacher (lector) and head of department in upper secondary schools for many years and as mathematics teacher educator (associate professor) for about fifteen years, first at Lund University and then at Kristianstad

University. My own education is a master of philosophy in mathematics (also with physics, theoretical physics, astronomy and pedagogy), a research degree in pure mathematics and an examination from upper secondary mathematics teacher education. As teacher educator since 1985 I have gradually grown into the field of didactics of mathematics, which did not exist in Sweden, when I took my early postgraduate education. The first courses in didactics of mathematics, which were organised at Lund University, took place in 1988 and I was part of the course team. My teaching has gradually moved from mathematics to mathematics education over the years and my research and supervision over the years have been in mathematics education.

### **The structure of doctoral programmes in the different countries**

In this section I want to offer short descriptions of some characteristic features of the doctoral programmes in didactics of mathematics in those Nordic countries where it exists. This includes some parts of the recent developments in each of the countries also.

#### *Denmark*

In Denmark the doctoral education lasts for three years and there is no compulsory course part. The doctoral student can have one or several supervisors. The research education leads to a thesis that will be examined by three examiners, and two of them are external. The examiners read the thesis and produce a written document, where they classify the work as acceptable or not. After the acceptance the doctoral student is allowed to defend the thesis in a public viva (disputation), where also external persons can criticise and discuss the content. The thesis is normally not published. At least five universities offer programmes which can include didactics of mathematics (at least as part of a broader subject description). They are Ålborg University, University of Southern Denmark, Roskilde University Center, Copenhagen University and Denmark Pedagogical University. Since 2004 Denmark has a National Graduate School in Science and Mathematics Education. The graduate school can offer financing for seven doctoral students and organises courses, seminars and workshops. The first professor of didactics of mathematics in Denmark was Mogens Niss and he has been followed by others recently.

#### *Finland*

Following some national and rather general guidelines, the doctoral education in Finland is designed in ways decided at each university. Models exist where different disciplines are integrated into the same programme as for example the sub-disciplines of pedagogy at Åbo Academy. There are also more separated models. The doctoral education is normally four years of full time studies and builds on a masters' degree. Most teacher educations in Finland are currently qualifying for doctoral education (Westbury, Hansén, Kansanen & Björkqvist, 2005). Teachers qualified to teach mathematics have either a master's degree in pedagogy (class teachers, years 1-6) or with the main area among mathematical subjects (subject teacher). No university offers a separate doctoral programme in didactics of mathematics, but such studies can be carried out under different examination rubrics. The following is an example from Åbo Academy, where courses and seminars take up about one year of full time studies in the doctoral education. Among them are common philosophical, theoretical and research methodological courses (about one semester). Normally the doctoral student has one supervisor, but a co-supervisor can be appointed if

needed. The content of the doctoral education is decided in an individual study plan, which is created by the student and the supervisor and accepted by the faculty board. Teaching experience is not a prerequisite for research studies in didactics of mathematics. The studies result in a thesis on licentiate or doctoral level (60 or 180 study points). Fewer students are now choosing to take the licentiate degree.

The dissertation is examined by at least two external experts, preferably with 'docent'-competency (see below). The examiners send their decisions to the faculty board, where they express suggestions for improvements and where they give a judgement of the acceptability of the work. The faculty board can ask for changes in the manuscript as conditions for a new examination. After the permission for defence of the thesis is given by the faculty board, a public disputas (viva) is carried out where the doctoral student presents the work and defends it. The official examiners, opponents and the public are given opportunities to discuss and criticize the work. The opponent has to hand in his written criticism to the thesis and the defence no later than a month after the disputas. Based on that, the faculty board decides about the judgement in an eight graded scale. Examiners can function as opponent at the disputas. All dissertations are printed (in paper or digital form). National graduate schools for teachers in mathematics, physics and chemistry in Finland were organised first during 1995 to 2001 and then again from 2003. It started with four doctoral students in 1995, then grew to 10 in 1998 and since 2007 it has five doctoral students. The responsibility for the graduate school lies at the University of Helsinki. The doctoral students must have a masters' degree in pedagogy or in any of the subjects involved. The main focus can be either pedagogy or any of the mathematical sciences. In both cases studies in subjects, in didactics of the subjects and methodology are included. The aspiration is to offer two supervisors for each student, one from pedagogy and one from mathematical sciences. Each year two five-day seminars for doctoral students are organised, with emphasis on research methodological themes. No doctoral courses are given. The graduate school has a close cooperation with The Finnish Mathematics and Science Education Research Association (Malinen & Kupari, 2003). Doctoral students are encouraged to present at its yearly symposia.

A distinct growth in number of dissertations in didactics of mathematics can be noted since 1995. During 1984 to 1996 6 dissertations were defended and during 1997 to 2006 it increased to 23 (Pehkonen, 2007).

In Finland there is a clear wish to develop further the cooperation between institutions in national and international graduate schools. A recent development plan for 2007 to 2012 is voicing such views (Undervisningsministeriet, 2007)

### *Iceland*

There is no special doctoral programme in mathematics education in Iceland. One student is at the moment doing her study in this field, but it is formally taken in a programme for Educational Studies – Philosophy of Education. Iceland has had a professor in mathematics education since 1992, but no research education has been created during that time. In general doctoral studies are based on a master's degree. The Study time is at least 3 years and at most 8. The course part can be of one year duration. A study plan is set up every year in collaboration between the student and the supervisors. The examination is done by two external examiners, who read the thesis in advance and write their comments. The student is allowed to make changes based on the comments within one month. The doctoral

candidate is then defending her thesis in a public meeting. It has been common for Icelandic students to go abroad to carry out their doctoral studies.

### *Norway*

In Norway the doctoral education is a three year study with course work and research leading to a written thesis. The course work in general is one semester but in the biggest programme (in University of Agder) the course work covers one year of study. The prerequisites for study are a master's degree in mathematics or mathematics education and teaching experiences. The examination is similar to the one in Denmark, with two external opponents and one local producing a written judgement in advance and then acting as opponents during a public defence (disputas). The dissertations can be published. There is no national Graduate School in Mathematics Education in Norway yet, but plans to start one. On the other hand, the Nordic Graduate School in Mathematics Education (see more below) is situated in University of Agder. University of Agder is at the moment the only university that offers doctoral courses in mathematics education at a regular basis in the Nordic countries.

Exemplifying with the programme of University of Agder, it can be noticed that each doctoral student will get at least two supervisors (a main supervisor and a co-supervisor) and an individual study plan is made up each year, followed by a yearly report to the board about the outcome of each study year. Also supervisors' reports to the board are handed in and carefully followed up. Two courses are compulsory, Theory of science from a didactics of mathematics perspective (5 study points) and Research methodology in Mathematics Education course (15 study points). One or two courses are running each semester and they normally attract doctoral students from the whole of Scandinavia.

### *Sweden*

The doctoral education in Sweden is a four year full time education building on a bachelor or masters' degree. The programme consists of courses and research resulting in thesis-writing. Normally the courses take up half of the time. It is possible to take a licentiate degree half-ways in the programme. This is an independent degree and the student can stop there. The normal case is to continue with the second part and that the two parts are closely linked to each other.

Programmes in mathematics education did not exist until fairly recently. The first ones were, as mentioned above, in Gothenburg, Luleå and Umeå. In Luleå one student graduated in 1996. The first one in Umeå graduated in 2001. In 2000 a national graduate school in didactics of mathematics was created, with ten participating universities. In addition to the mentioned ones they were: Kristianstad, Linköping, Mälardalen, Stockholm, Uppsala, Växjö and Royal Technological University (KTH). The graduate school was funded from Riksbankens Jubileumsfond (Swedish Bank Centennial foundation) and Vetenskapsrådet (VR, The Swedish Research Council). In 2001 21 doctoral students were taken up at the ten different institutions and were intended to finish in 2006 (studying on 80 %-basis for five years with a 20 % teaching duty). In 2006 eight of them defended their theses, one finished in 2007 and the rest of them are still carrying on with their studies (delayed by pregnancies or sick leaves).

The graduate school finished to exist in 2006 (a one time funding) and an application for a continuation has been sent to VR but has not yet succeeded.

The programmes in Sweden are situated at mathematics departments and doctoral courses in mathematics are part of the course work. The prerequisites were three semesters of fulltime mathematics studies and mathematics teacher education or equivalent experiences. In the national Graduate School it was supposed that students take 50 % courses in mathematics but most of them actually did less than that. Courses in general pedagogy or mathematics education dominated. One or two supervisors work with the student and individual study plans are created each year. In the national graduate school there was a close follow up of the students and they were given a special mentor (from the board of the school) in addition to the supervisors. The graduate school offered common courses in mathematics education, and seminars and workshops. The courses in mathematics were supposed to be the normal ones given at the department for doctoral students in mathematics. The graduate school also tried to offer competence development for the supervisors, but these arrangements were not used much by the possible participants (Leder, Brandell & Grevholm, 2004).

#### *The Baltic countries, Estonia, Latvia and Lithuania*

At the moment there is not a separate doctoral programme in mathematics education in Estonia. Mathematics education is considered as one specialisation in the doctoral programme in general pedagogy. Such programmes are offered in Tallinn University and University of Tartu in cooperation with faculties of education and mathematics. In general, the doctoral education is a four-year study with course work and research. The course work (courses in general pedagogy and didactics of mathematics) covers one year of study and is compulsory. The research leads to a thesis that will be defended during a public defence and judged by two opponents (at least one external). The dissertations are published. The prerequisites for study are a bachelor degree in mathematics (3 years of studies) and a master's degree in mathematics education (2 years of studies). The situation in other Baltic states, in Latvia and Lithuania, is similar.

### **Collaboration in Graduate Schools**

#### *National graduate schools*

Research areas that are small with only one or two students and one or two faculty are vulnerable and it is tempting to create cooperation between institutions. As can be seen from above the idea to build National Graduate Schools has developed in Finland, Sweden, and Denmark. Finland was first in 1995, followed by Sweden in 2000 and Denmark in 2005. Finland has repeated the initiative once, in Sweden it is so far a one time activity between 2000 and 2006. Many reasons have been presented for having national graduate schools. There is a wish to increase the number of students finishing in time, a wish to shorten the actual study time (which normally can be longer than the formally expected time), to offer a richer study environment for the students and to offer competence development for the supervisors. In Sweden an evaluation of 16 national graduate schools was published in 2006 (Persson, 2006). The report points out that there have been some problems, such as lack of knowledge about and experiences in didactical research, inadequate planning and organisation of the activities, in some cases insufficient supervisor competence, difficulties to cooperate and antagonism between different fractions in the subject fields, financing problems and so on. The evaluation of the outcomes is more positive. Most doctoral students have finished in reasonable time and supervisors'

competence has developed well. National and international networks have been established and are strengthening the opportunities for further development and improvement of research in subject didactics. The relations to the core subjects have been highly improved. There seems to be a promising labour market for the new doctors. Person points out that when establishing new graduate schools these experiences must be taken into account. There must be adequate supervisor's competence from the beginning and resources for competence development of supervisors must be set aside. Not too many institutions should be involved and very good preparations are necessary for a graduate school to function well from the beginning. All participating institutions must agree about the aim and goals of the activities. Common guidelines for students' conditions and financing must be agreed upon.

### **The Nordic Graduate School in Mathematics Education**

In 2004, the Nordic Graduate School in Mathematics Education started, based on funding from the Nordic Research Academy (NordForsk). It is a five year activity with the idea that after five years the cooperation built is strong enough to survive by support only from the involved institutions.

I will present the Nordic Graduate School in Mathematics Education, its aims and some of its activities. The Graduate School is a network of about 40 Nordic and Baltic research environments with graduate education in mathematics didactics. Around 115 supervisors and 86 doctoral students are part of the network. An account will be given of doctoral courses so far, of seminars for supervisors and of workshops and summer schools that have taken place.

#### *The aim of a Nordic Graduate School in mathematics education*

The aim of the Nordic Graduate School as it was decided by the application to NordForsk in 2003 (The Nordic Research Academy) is to

- support and develop the education of researchers in mathematics education in the Nordic and Baltic countries,
- create constructive cooperation in order to raise the scientific quality of research in mathematics education,
- give all doctoral students in mathematics education access to the activities of the Graduate School
- create cooperation among a greater group of doctoral students and supervisors in order to share experiences and opportunities to improve the education of researchers.

The utmost aim is to create a network of cooperating partners, who can continue to collaborate after the five years of the Graduate School (Grevholm, 2004a).

#### *Activities in the Nordic Graduate School in Mathematics Education*

The activities in the Graduate School can be summarised in the following points (Grevholm, 2004b, 2005a):

- Common courses are created with the added competence from all researchers in the Nordic countries and international partners (Grevholm, 2004c)

- Seminar-series in specific research areas are offered as a complement to local series and workshops on subjects or issues of main importance (Grevholm, 2005b)
- Competence development for supervisors and exchange of experience is offered
- Partnerships and collaboration with distinguished international scholars are built
- Creating a database for ongoing work, theses and greater development work in mathematics education
- Mobility stipends and special financial support for doctoral students are given.

#### *Courses that have been offered since 2004*

The courses offered are of two kinds. Courses that are given on a regular basis at some of the participating universities are open to all doctoral students in the network. They are advertised each semester. Other courses are initiated by the board of NoGSME. The board collaborates with some interested colleagues in one of the participating universities and the course is constructed and given at that place, with financial support from NoGSME (Grevholm, 2004d, 2005c). The regular courses so far have been given at University of Agder in Norway. The courses that have been initiated by NoGSME have taken place in Copenhagen University (Winsløw, 2006), Denmark Pedagogical University, and Norwegian University of Technology. Ongoing courses are planned together with Roskilde University and Umeå University. Here are the courses given so far or ongoing:

- Theory of science from a mathematics education perspective
- Meta-perspectives on mathematics and the learning of mathematics in a technological environment
- History of mathematics with emphasis on modern mathematics
- Theoretical aspects of mathematics education with emphasis on the French School
- Problem solving in mathematics education
- Theories of learning and teaching mathematics
- Research design and research methods in mathematics education
- Views of knowing and learning: Constructivism and socio-cultural theory
- Gender and mathematics education
- Justification of research in mathematics and science education with special emphasis on the role of theory in such justification
- Research on assessment in mathematics education

Students get travel support to come to the courses and they can also apply for mobility stipends if they want to spend one or two months at another Nordic university. The mobility stipend covers real costs for travels and accommodation.

#### *Summer schools*

In 2004 four students and the director of NoGSME took part in the ERME summer school in Podebrady in order to get inspiration for creating the coming summer schools of NoGSME (ERME is the European Society for Research in Mathematics Education). These four students were then part of the planning group for the summer school in 2005. That year 30 students took part in the NoGSME summer school in Finland in Jyväskylä University, which according to the evaluations of the participants was a great success. In

2006 20 students took part in the NoGSME summer school in Dømmesmoen at the University of Agder campus in Grimstad in Norway (Grevholm, 2005ab, 2006b). Again two doctoral students were part of the planning group and helped to make it fruitful to their colleagues. The main part of the programme is taken up by work in groups, where each student can get her research project discussed and commented on. The groups are lead by international experts in the field, which is highly appreciated by the participants. Among these experts we have had Gilah Leder, Kath Hart, Tommy Dreyfuss, Marcelo Borba, Abraham Arcavi, Eva Jablonka, and Marianna Bosch. In 2007 NoGSME offered a summer school in Laugarvatn in Iceland, which had 42 participants and in 2008 NoGSME the summer school will take place in Sweden. The friendship and mutual understanding that is built in these summer schools are expected to be the foundation of longstanding cooperation of the students in their coming careers (Grevholm, 2004b, 2006b).

### *Seminars for supervisors*

A crucial component of doctoral education is access to good and experienced supervisors. In order to assist the environments in strengthening the competence of supervisors NoGSME is organising seminars and competence development programmes for supervisors. They have focussed much on quality issues in research education and publications (both papers and theses) (Grevholm, 2006c). NoGSME has built a close cooperation with the journal Nordic Studies in Mathematics Education (Nomad) in order to enrich these programmes. Here is a list of the seminars given so far:  
September 2004 in Vasa, Quality in research in mathematics education  
April 2005 in Korsør, Quality of theses in mathematics education  
September 2005 in Trondheim, Supervision of doctoral students  
November 2005 in Lund, Reviewing of papers in mathematics education  
May 2006 in Vasa, Research programmes in mathematics education  
October 2006 in Magleås, Critical situations in supervision of doctoral students in mathematics education  
February 2007, Trondheim, Review process of papers for scientific journals  
October 2007, Lund, Outcomes of research in mathematics education.

The seminars most often have between 20 and 30 participants and quite an important network of researchers is growing from the meetings that take place there. International scholars have been invited and generously offered from their expertise. Some of the invited researchers so far have been Frank Lester, Diana Lambdin, Uri Leron, Erkki Pehkonen, Gunnar Gjone, Carl Winslöv, Morten Blomhøj, Paola Valero, and Barbara Jaworski.

### *Workshops*

NoGSME organises workshops on central research issues of interest for the participants in the Graduate School (Grevholm, 2007a). The activity involves both doctoral students and supervisors. The first took place in September 2005 in Trondheim and the workshop dealt with classroom research and was led by Simon Goodchild. The second workshop focussed on research on mathematics textbooks and was in May 2006 in Kristiansand. The experts here were Birgit Pepin and Linda Haggarty. Here a Nordic network for research on mathematics textbooks was created. In November 2006 in Kristiansand a third workshop on research on use of ICT in mathematics education took place with 25 participants and two invited experts, Luc Trouche and John Monaghan. The fifth workshop was on

mathematics and language and it took place in Uppsala in Sweden in April 2007 with Heinz Steinbring and Candia Morgan as invited guests. In November 2007 a workshop on Justification of research in mathematics and science education with special emphasis on the role of theory in such justification, lead by Mogens Niss will take place and it is closely linked to the corresponding doctoral course. Patricio Herbst will be one of the invited lecturers.

#### *The board of the Nordic Graduate School in Mathematics Education*

The board consists of the director, one member from each of the five Nordic countries and a representative for the Baltic countries. Board members currently are Barbro Grevholm, director, Christer Bergsten, Sweden, Trygve Breiteig, Norway, Ole Björkqvist, Finland, Gudny Gunnarsdottir, Iceland, Madis Lepik, Estonia, and Mogens Niss, Denmark. The members of the board are not paid for their work, but contribute for idealistic reasons and as part of their positions at the home university. The board meets about three times a year in connection to other NoGSME- activities. The board is responsible for the initiatives and work and has to report to The Nordic Research Academy once a year. Most of the board members are also active in their national society for research in mathematics education and in national graduate schools.

#### *Cooperation with Nomad*

NoGSME has close cooperation with the journal Nomad, Nordic Studies in Mathematics Education. Doctoral students and supervisors are invited to publish in Nomad and in each issue of Nomad a few pages are devoted to the NoGSME programme and activities (Grevholm, 2006a). Here they can publish in their Scandinavian mother tongue or in English.

#### *International centres of excellence are working partners*

To get support for the application to NordForsk in order to get financing for NoGSME we turned to a number of important international centres of excellence and asked them to write letters of support for us. Leaders from these centres have then been involved in our plans and activities in different ways. The centres we collaborate with are Institute of Advanced Study, La Trobe University, Gilah Leder, Concordia University, Anna Sierpiska, University of Michigan, Hyman Bass and Deborah Ball, University 7, Paris, Michele Artigue, and University of Klagenfurt, Didaktik der Mathematik, Willibald Dörfler. Another important discussion partner has been Jeremy Kilpatrick, who is well informed about Nordic conditions relating to mathematic education. He has among other things been a guest professor at Gothenburg University and the supervisor of some Swedish doctoral students.

#### *Results and outcomes of the Nordic Graduate School*

The activities of the Nordic Graduate School are building strength in Nordic research for the future (Grevholm, 2006e). The knowledge and contacts that doctoral students and supervisors are getting from the events together offer insights that can not be achieved from reading books or by other means. In the future these links will be important and valuable for the field of mathematics education. Models of organising research education and supervision can be compared and developed and fruitful ideas from one university can

be spread to other places (Grevholm, Persson & Wall, 2005). It is especially important for the Nordic Graduate School to build the contacts with colleagues in the Baltic countries.

### **Some features in order to strengthen the quality of researcher education**

#### *Ninety percent seminars*

Mathematics education as a field of research is developing in the Nordic countries but it is still a young area and there is a need to assure the quality of the work and to live up to international expectations and standards. A number of initiatives have been taken in order to raise quality. For example, both in the Swedish Graduate School and at UiA in Norway we have introduced what is called ninety percent seminars. This means that when the student and supervisors agree that there is a manuscript of about 90 % of the final thesis a seminar is organised. To this seminar an international scholar, who is expert in the area of study, is invited. He or she reads in advance the 90 % finished manuscript and gives constructive and creative feedback during the seminar, which is organised as a dissertation. The intention of the seminar is to inspire the doctoral student to raise quality in the final phase of writing and to get fresh ideas how to improve the dissertation and to be aware of possible criticism before it is too late. The seminars have proven to be of utmost value to both the doctoral students and the supervisors. International scholars have generously given from their expertise in these discussions.

#### *International studies*

Another feature of importance for quality is international collaboration and studies abroad. There is an expectation for the students to spend one semester at another university, thus learning about a different academic institution and meeting other mentors and supervisors. This has functioned in Sweden, where the programme is often taken over five years but has been difficult to realise in Norway within a three year programme. The students feel the time pressure too hard for going away for such a long period of time. As compensation we have invited many international scholars to give seminars at UiA, but this is of course not the same as spending time abroad. We are working on how to improve this feature of the education. Internationalisation is also a concern of the Norwegian and Swedish educational authorities (SOU 2004:27).

#### *Models for supervision*

Supervision is a crucial part of the doctoral education. In order to ensure good and continuous quality in supervision we have at UiA decided to have at least two supervisors. Supervisors move, get sick or retire and it is important that the students are not left in an unstable situation. Joint supervision and other forms for organising supervision must be considered. At Luleå University of Technology a dynamic model of supervision with many levels have been used and proven successful (Grevholm, Persson & Wall, 2005). The model mirrors an apprenticeship theory for the doctoral education, which seems to be embraced by many of the supervisors.

#### *Public defence of the dissertation*

A public defence of the dissertation and invited international opponents is typical of the Nordic doctoral educations. It seems very important to have open discussions, where anyone can question and criticise the dissertations. Also the publication of theses, which makes them accessible in libraries to everyone, is valued in the democratic Nordic

societies. The publishing of thesis is the normal situation in Finland and Sweden and often is the case also in Denmark and Norway. Nowadays in addition to the printed books with theses there is often also an electronic version on the internet.

#### *A Nordic Journal for Mathematics Education*

The close collaboration with the journal *Nordic Studies in Mathematics Education*, *Nomad*, is of great value to supervisors and doctoral students in the Nordic countries. This journal is the natural choice for the first publications of the students. But many of them prefer other international journals as *ESM*, *JMTE*, *IJSME* or *FLM*.

### **Crucial or critical issues for mathematics education doctoral programmes in the Nordic countries**

#### *Supervision in a new research field*

Trying to build up and expand a new research field is not an easy task. The most problematic issue has been that there have not been many experienced researchers, who can function as supervisors. In Sweden, for example, many mathematicians accepted to be supervisors when the national graduate school started. Some of them realised that they could only be of help for general matters in the education and someone else had to do the actual mathematics education supervision. But others actually thought that they had the expertise (being expert mathematicians but amateurs interested in teaching and learning of mathematics). Thus over the years there has been a number of situations, where the board of the graduate school had to assist in finding new supervisors, often by using an international scholar as additional supervisor. Also it happens that the student and supervisors are not getting along in a good way and a shift of supervisor has to be made. This is difficult when not many choices are available. Thus some supervisors have been used to an extreme extent over some years.

As the access to experienced supervisors was limited there was a need to build competence. This has been tried both in the Swedish Graduate School and in the Nordic Graduate School. The success was limited in the first case because of lacking interest among the group of supervisors. In the Nordic Graduate School it seems to work well. The education of new supervisors is crucial for future survival of the area and we are focussing on getting all the new doctors to participate, thus fostering the future generation of supervisors. The quality of supervision is critical for the outcomes and here international contacts and links are of extreme value.

A first national conference on supervision of doctoral students was held in Sweden in 2003 and some research has been carried out in this area (Strömberg, 1979; Strömberg Sölveborn, 1983; Lindén, 1998). The international community in mathematics education has also cared for the issue of supervision (Hart & Hitt, 1999; Leder, 1995).

#### *Intersubject collaboration*

Collaboration between researchers in mathematics, mathematics education and general education has been tried in all the Nordic countries with varied success. In the beginning of the Swedish Graduate School there seemed to be a mini Math War going on. Later this faded away, probably because the mathematicians realised that what was going on is not dangerous for them, on the contrary. This development is even visible in the evaluation of the graduate schools in Sweden (Persson, 2006).

### *Issues of format and language in theses*

The format of the thesis – monograph or selection of papers with preamble (“kappa”) has been much discussed in the graduate schools. The tradition from pedagogy is to write monograph and from mathematics it is a selection of published papers with a preamble. As most of the students have been situated in mathematics departments they have been strongly influenced to write a selection of papers. From the 9 in Sweden finished so far there is only a few strict monographs. One wrote the licentiate thesis as a monograph and the second part of the thesis as a selection of papers. Another discussion is how many of the papers must be published in journals before the dissertation. In mathematics there has been a development towards accepting theses where none of the papers are published. So there has also been shifting traditions in mathematics education. One of the Swedish students had 6 published papers in the thesis and others had only two or three non published papers. Another critical issue is the question of language for the dissertation - mother tongue or English? In Sweden there has been a public debate about scientific papers written by Swedes in bad English. They are claimed to make fools of themselves internationally. It is obvious that almost every non-native English speaking writer is much better in expressing fine nuances in the mother tongue than in English. But it is also clear that writing in English opens for international readers. And later on researchers must write papers in English anyway. Not using mother tongue leads to a poor scientific language in the local languages and publications that will not be read by teachers in school. There are many pros and cons to consider before the decision on language is taken. In the end it is up to the student and the supervisor and must be taken in each specific case taking care of the circumstances for each student. A student who has writing difficulties anyway will have still worse problems if the writing is in English.

### *Financing during and after the dissertation*

The sources for financing doctoral studies differ from one place to another. In Sweden and Norway the student must have guaranteed financing for the studies before he or she can be taken up in a doctoral programme. The state offers a number of doctoral positions and there can be positions inside specific research projects. The student is employed by the university for 3-4 years and has legal rights as employee. The salary can be compared with that of a beginning teacher. After the dissertation the position is finished. There is a lack of post doctoral positions in didactics of mathematics and this creates problems for those who want to go on at once after the dissertation and qualify themselves to become a docent. In Sweden and Finland this is an academic title for which one must qualify through research and publications after the doctoral degree (the same as Habilitation in Germany). The normal rule of thumb is to publish as much as a second thesis. An application must be made to the faculty and the scientific work is evaluated by external international experts and a public popular scientific lecture is given and evaluated by a scholar in another research field. Based on these activities the decision is taken about becoming the docent title. In Sweden the main supervisor of a doctoral student must be at least on the level of docent. In the Nordic countries academic studies are free, no costs are paid by students but all is paid by tax-money. Thus the salary of a doctoral student can be used entirely for the private consumption.

### *Vulnerability of small research environments*

Another critical issue is the fact that many research environments in mathematics education in the Nordic countries are small with only one or two faculty and one or two students. It is difficult to solve the supervisor problem and to create a vivid and inspiring work situation in a community of researchers. One solution for this situation is collaboration between two or more institutions or to be part of a graduate school. The evaluations indicate that graduate schools are efficient in offering what the student needs as a complement to a small environment (Persson, 2006).

#### *Opportunities to finance collaboration in graduate schools or Nordic networks*

Collaboration in networks of graduate schools is rewarding and helps to assure quality. But there must be financial resources for such work. In Finland the graduate school succeeded in getting a continuation but in Sweden so far this has been unsuccessful. It is critical to find opportunities to solve this problem. The research environments that have been built up during the time of the graduate schools can very easily be torn down again if there is no continuation of collaboration.

#### **Some questions raised in relation to the conference on doctoral programs**

*What variations are there in the program beyond the master's degree, and how are those variations taken into account?*

In the Nordic countries it is not possible to take a Ph D only by course work. The research part is the essential part, and must take up at least 2 years of the time spent in the program. As mentioned above in Finland and Sweden it is possible to take a licentiate degree half way through the program. In Finland and Sweden an academic career expects the doctor to qualify for docent through published research.

*What is your experience with cross-institutional, as opposed to single-institutional, programs?*

In collaboration with departments abroad we have very good experiences from student exchange. The evaluations show that students' experiences gained during a stay at another university improves the quality of the education. It has been easy to find colleagues abroad willing to take a guest student and provide support and access to courses, seminars and communities for cooperation.

No problems have been experienced, when different departments have give courses to students from several Nordic universities. Colleagues with expert knowledge in a specific area have been willing to create courses in collaboration with the Nordic Graduate School with just a little extra financial support. Thus the best from many institutions have been made available for all doctoral students in mathematics education. Evaluations indicate that this is highly important for the students.

In some cases students have realised that the supervision they get at home is not as good as what other students get. Such cases have initiated actions and initiatives for improvement.

*How is the program evaluated?*

Any university can create their own doctoral programs, but university colleges in Sweden are not allowed to have own programs. In Norway they can if they get them accepted by the National Committee for Evaluation of and Quality in Education (NOKUT).

In Sweden academic subjects are evaluated regularly both at basic level and higher levels. This is done by external international experts and published reports are produced.

In the UiA program we are discussing changes as our dean wants us to have one semester of course work and the teachers want two. They argue that the interdisciplinary character of didactics of mathematics demands some common knowledge in the area. An external evaluator will soon be appointed for the program at UiA after its first five years.

The most influential evaluation is probably the one which is made by students taking the program. If students are satisfied with the program this is made known widely by themselves.

Quality assurance systems have been implemented in the Nordic countries. The process includes among other things regular evaluations made by participating students. Courses are evaluated in mid-term and at the end of the course. Each student has an individual study plan to follow, agreed between the student and the supervisors. The plans are followed up carefully by the board of the program. Students hand in yearly reports on their progress to the faculty board. Supervisors hand in yearly reports on students' progress and supervision. A lesson learnt so far at UiA is that students consider 3 years to be too short and that one year of courses is too much in addition to participation in research projects.

The Nordic communities in didactics of mathematics seem to be equal proportions of female and male students. Among supervisors there is an overweight of male academic teachers. The Nordic professors in didactics of mathematics were male dominated until 2003 when suddenly four female professors were appointed. Another additional female professor in 2007 almost creates gender balance in this small Nordic group.

*What is the importance of knowledge of mathematics by doctoral students in your program?*

The fact that the program is situated in a mathematics department indicates that mathematics plays an important role. A solid foundation of mathematics must be part of the bachelor and masters education that forego the doctoral education. In the Swedish Graduate School the emphasis on mathematics was still greater as doctoral courses in mathematics were a substantial part of the coursework.

*What is the importance of knowledge of technology by doctoral students in your program?*

In the methodology course students become familiar with software for both quantitative and qualitative data analysis (for example SPSS, NVivo, Atlas, digital video recoding and tape recording) and they are using many different programs in the writing of the thesis (for word processing, drawing, diagrams, reference management, layout). Most students have a very good background in technology when they start in the program, where they get access to all software they need and can get special courses for it. In their data collection in schools they meet most of the programs that are used in mathematics teaching and learning (Excel, Derive, Mathgraph, Cabri and so on) and have often used them themselves as teachers of student teachers.

*What have I learnt during the conference?*

Some issues and questions discussed during the conference are also crucial in the Nordic countries, such as the structure and content of a program and challenges and opportunities of the delivery of the program, prerequisites, recruitment and progress of students. The role

of mathematics in a didactics of mathematics program is important also to us. The eternal question of what is a good enough thesis is common. The question of accreditation is not one we have discussed, probably because of different academic traditions and culture. The issue of how a program can supply teaching experience is not one we would raise. The doctoral education in the Nordic countries is seen as an education to become a researcher. The education to become a teacher must be given as a separate option (and is normally of equal length as the doctoral education). All academic teachers must take a course in university didactics or pedagogy.

Some of the issues that have taken up our considerations in the Nordic countries are how to create and supply excellent supervisors and the questions about quality in research, publications and theses. In the conference I did not notice much concern about supervision? Is that because mathematics education has been established so long in the US that it is evident that there must be access to good and experienced supervisors? In the Principles to guide the design and implementation of doctoral programs in mathematics education there is nothing about the relation between the doctoral student and the supervisors (AMTE, 2003).

Another aspect that became more transparent to me during the conference is how unique and valuable the cooperation between programs in the Nordic countries is. That is one feature the US-programs could learn from. This was noticed by Alan Bishop already in the first conference on doctoral programs. He wrote: "To me this situation cries out for inter-institutional collaboration, making the best of each university's faculty and programmatic resources available beyond its usual enrollment boundaries." (Bishop, 2000, p 59)

Aspects of quality have obviously been of interest also to the community in US (see for example Lester & Lambdin, 2003; Schoenfeld, 2003) and in the Nordic community we have profited from earlier discussions and experiences that were built up by the US-colleagues in the field.

### **The future of doctoral programmes in mathematics education in the Nordic countries?**

Do we have a critical mass of researchers in order to keep the activities alive? How do we ensure quality and endurance of programs? What opportunities are there to improve the programs and in what ways?

Is there a need for more persons in the labour market with a doctoral degree in mathematics education? Do we need research on doctoral education in mathematics education? Will society continue to ask for research in mathematics education?

There are many questions to inquire into and try to answer about postgraduate education in the Nordic countries (Grevholm, 2007b). The cultural and social conditions are similar in the five countries and problems are often the same. Also solutions seem to be similar and the public debates have parallels.

The government in Sweden has shown great concern about the research education. It was restructured in 1998 and a first evaluation of the results was published in 2007 (Högskoleverket, 2007). One outcome is that the students that graduate within a period of five years have increase from 16 to 28 % of the population. The number of degrees has increased with 50 % after the reform and stays at that level. In 1990 0.6 % of the working population (between 25 and 64 years of age) had a doctoral degree and that increased to 1.0 % in 2005. An investigation in 2002 took care of specific questions about the doctoral time

and the time after graduation (SOU 2004:27). Doctoral education has expanded with 100 % between 1990 and 2000. In Sweden the number of doctoral students is about 13000 (fulltime equivalents). It would be astonishing if there were no problems in such a strongly expanding activity. A large generation of persons born in the 40ties are in the process of retiring and the new academics with a doctoral degree seem to have a prosperous labour market to enter into. As mathematics knowledge is seen as one of the tools a citizen in a modern society will need it seem probable that questions about teaching and learning mathematics to still larger groups of the population will be in focus. Most governments realise that we are moving into an international society, where the human capital resources in the form of education and competence are the means to survive and compete internationally through excellence and growth.

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