

A REVIEW OF THE RESEARCH ON THE ROLE OF EXTERNAL REPRESENTATIONS ON UNDERSTANDING AND LEARNING MATHEMATICS AND PROBLEM SOLVING

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ABSTRACT

Nowadays the centrality of external representations in teaching, learning and doing mathematics seems to become widely acknowledged. There is strong support in the mathematics education community that students can grasp the meaning of mathematical concepts by experiencing multiple mathematical representations. Two different classes of external representations are distinguished in the literature: descriptive and depictive representations (Schnotz, 2002). Descriptive representations consist of symbols that have an arbitrary structure and that are associated with the content they represent simply by means of a convention, i.e., text, whereas depictive representations include iconic signs that are associated with the content they represent through common structural features on either a concrete or more abstract level, i.e., visual displays.

A visual representation that plays an important role in the teaching of basic whole number operations is the number line (Gagatsis & Elia, 2003). However, findings of recent studies have shown there is a mismatch between young students' understanding of whole number operations and their understanding of the number line model of relevant operations (Gagatsis & Elia, 2003). Difficulties in using the number line may be due to its dual nature: According to Gagatsis, Shiakalli and Panaoura (2003) number line constitutes a geometrical model which involves a continuous interchange between a geometrical and an arithmetic representation.

Research in mathematics education discusses also the importance of using multiple representations in the problem solving process. A study conducted by Gagatsis and Elia (2004) accomplished the development and empirical test of a theoretical model verifying and interpreting the significance of the use of different modes of representation and more specifically decorative pictures, informational pictures, number line (depictive representations) and verbal description-text (descriptive representation) in solving addition and subtraction problems. It was found that students of Grades 1, 2 and 3 dealt with problems in verbal form and problems accompanied with number line or decorative picture in a similar and consistent manner. In other words, students overlooked the presence of the number line or the decorative picture and gave attention only to the text of the problem. As for the function of informational pictures in problem solving, it was found to differ significantly from the use of the other forms of representation. Students dealt in a unique and less flexible way with these problems. It is possible that it required extra and more complex mental processes relative to the other modes of representation.

Diagrams are also considered as a kind of visual representations which may be used as a powerful tool for mathematical thinking and problem solving. In problem solving a diagram is focused on the representation of the structural characteristics and not the surface details of the problem. Pantziara, Gagatsis and Pitta-Pantazi (2004) explored the role of diagrams in non routine-problems by 12-year old students. The results of the study showed that diagrams facilitated the solution of the problems for some students, while they caused difficulties in the solution procedures of other students. Thus, a specific diagram does not have the same impact on all students. In order to use diagrams effectively, students must develop the ability to translate the word problem into a diagrammatic representation and the ability to interpret a diagram in terms of a given word problem (Diezmann & English, 2001).

A number of studies investigated the contribution of the use of external representations on mathematical thinking and learning by students from secondary and tertiary education (e.g., Evangelidou, Spyrou, Elia, & Gagatsis, 2004; Gagatsis, Elia, & Andreou, 2003; Gagatsis & Shiakalli, 2004). Gagatsis, Elia and Andreou (2003) conducted a research examining a possible compartmentalization of the modes of representation of the mathematical concept of function (i.e., graphic, symbolic, verbal) by students of grade 9 and grade 11. Compartmentalization is an issue which requires careful attention in the learning of mathematical concepts by using multiple representations. The existence of compartmentalization reveals a cognitive difficulty that arises from the need to accomplish flexible and competent translation back and forth between different kinds of mathematical representations. Gagatsis, Elia and Andreou (2003) found that different types of translations among representations of the same mathematical content were approached in a completely distinct way by students. Thus, understanding of one representation or translation of a mathematical concept did not necessarily imply the understanding of another one. This behaviour indicated that students did not construct the whole meaning of the concept of function and did not grasp the whole range of its applications. As Even (1998) supports, the ability to identify and represent

the same concept in different representations, and flexibility in moving from one representation to another, are crucial in mathematics learning, as they allow students to see rich relationships, and develop deeper understanding of concepts. Furthermore, the results of the study of Gagatsis and Shiakalli (2004), which investigated university students' translation ability for the concept of function, indicated that the ability to translate from one representation to another should be considered as an important factor in problem solving.

In another study, carried out by Evangelidou et al. (2004), the different types of representations of function, that second year students of the Department of Education at the University of Cyprus could handle, were found to be closely related to the definitions for function and examples of functions' applications in real life situations, that they gave. These three factors (giving definitions, providing examples and handling different representations of functions) seemed to contribute critically in their own unique way to students' acquisition of the complex concept of function.

References

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