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### **Use of ICT in school mathematics.**

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The issue of this presentation is change of content linked with the use of ICT/CAS in upper secondary mathematics. It is a common experience from my own (Andresen 2006) and others' research that the use of ICT/CAS is commonly linked with teaching that favours:

1. Problem solving
2. Numerical modelling and solution and qualitative illustrations
3. Technical aspects

on the expense of proof- and theory issues. This was for example the case in the *World Class Math and Science* project, which encompassed a subproject on CAS use in mathematics (Andresen 2006 p 28). The research project offers a concrete example of change of content, compared with more classical upper secondary teaching: Learning differential equations with the use of 'Derive'<sup>1</sup>.

- Numerical methods for solving Differential Equations (DE) provides for mathematical modelling released from traditional restrictions in the form of request for analytical solution
- Exploration of DE models supports the students formation of conceptions like solution, families of solutions, equilibrium point etc.
- The interpretation in a suitable notion of the students' explorative work serves to point out crucial elements of their modelling activities, which are facilitated by the use of computer routines.

The use of CAS was prescribed as one amongst other innovations. So, recent developments in upper secondary school mathematics in Denmark were

- The use of CAS in parts of the written examination
- A number of multi disciplinary projects and activities
- Introduction of new forms of writing in mathematics: reports, synopsis etc

In total, after the reform of upper secondary, we have in mathematics:

- Teachers with a high degree of professional autonomy
- A rather detailed syllabus corresponding to the written examination
- Spare time for optional themes, multidisciplinary projects etc (app. 25% of the time)

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<sup>1</sup> Derive from Texas Instruments is a system for doing symbolic and numeric mathematics on PC. It processes algebraic variables, expressions, equations, functions, vectors, matrices and Boolean expressions like a scientific calculator processes numbers.

The earliest effects of the reform may now be entangled, even though the reform has still not been profoundly evaluated. We now see tendencies towards to change the content of multidisciplinary activity compared to single– discipline activities. The mathematical content of reports and documents from the multidisciplinary activities tends to be:

- Tools for graphic illustration, calculations and/or statistics
- About mathematics history, philosophy, culture etc.

These issues were seldom dealt with, before the reform.

All in all, it seems that the reform’s change of focus in mathematics will include the following two trends:

- Increased focus of attention on mathematics as a tool for calculation, estimation and concrete modelling and problem solving
- Broadening out the field of study within school mathematics

What will happen, then, to mathematics ’itself’?

This question cannot be answered without to discuss what is meant by the terms mathematics itself! In the following, I take the viewpoint of Realistic Mathematics Education, that mathematics is a human activity concerning with horisontal and vertical mathematising. Together with a colleague, I am working with one way to combine two models for learning and reflection (Andresen and Froelund 2008). The first model is Gravemeijer’s model (fig 1). Each rise from one level to the next in this model of mathematical activity can be supported by reflections at a corresponding level. The second model is a stratification of mathematical reflections (Neubrand 2000). The basic idea of our work is to put a stronger focus on mathematics itself in the sense of mathematising, by the means of a stronger focus on mathematical reflections. So, we suggest a means to balance the teaching of mathematics after the reform by developing a model that aims to:

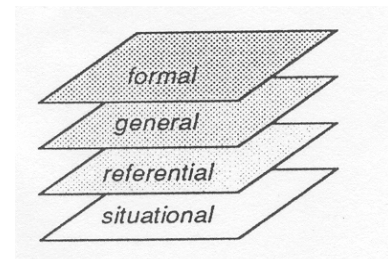


Fig 1. Levels of activity.  
Gravemeijer, K. & Stephan, M.  
(2002). p.159

- Support the content of learning and doing mathematics
- Develop the students’ mathematical competencies and profound understanding of mathematical activity and theory
- Strengthen the students’ ideas of what mathematics is about

About mathematical reflections:

- Reflections presupposes and supports the development of intellectual autonomy
- Reflections cannot be imposed on the student – they must take their free course
- Reflections may be guided by the teacher, be inspired by thought-provoking questions

Our model for teaching that focus on mathematical reflections prescribes that the teacher has to prepare and use a so-called reflection guide. The aim of preparation and use of the reflection guide is to:

- Help the teacher to be conscious about and to use thought-provoking questions about mathematical thinking
- Structure the teacher's analysis of students' learning trajectory
- Give weight (by the students and the teacher) to the core perspective of mathematics as an activity and, thereby, to give a more balanced view on mathematics

We have outlined guidelines for the preparation of a reflection guide. The overall preparation may happen in four steps:

- Choice of a suitable part of the students' imagined learning trajectory
- Design of the structure of inclusion in the teaching sequence
- Identification of potential levels of activity in the sequence and/or in the materials
- Formulation of thought-provoking questions

followed by a more detailed planning.

The rest of this presentation gives an example of thought provoking questions at four levels from a reflection guide, tailored for use in connection with teaching based on the text in (Fig 2). The text is my translation of an excerpt from a booklet on differential equations from a modelling perspective.

ENCL III: Let us now try to solve the differential equation

#14:  $\frac{dp}{dt} = kp$

As a model for unbounded growth of a population with  $k = 0.04$  and  $P(0) = 4.6$ , we obtain

#15: DISOLVE(0.04, p, -1, t, p, 0, 4.6)

Which gives

#16:  $25 \cdot \ln(p) - t = 25 \cdot \ln\left(\frac{23}{5}\right)$

And therefore

#17: SOLVE(25 · ln(p) - t = 25 · ln(23/5), p)

#18:  $p = \frac{23 \cdot e^{-t/25}}{5}$

OR

#19:  $p = 4.6 \cdot e^{0.04 \cdot t}$

Hence, we have exponential growth. The graph is below.

It is common to use "neutral" designations like  $x$  for the independent and  $y$  for the dependent variable. The differential equation that describes the unbounded growth of a population then turns to:

#20:  $\frac{dy}{dx} = ky$

To which the complete set of solutions are determined in the next paragraph a)

The temperature of coffee in a cup changes over time and experiments have shown that the rate of change of the temperature  $h$  is proportional with the difference between  $h$  and the temperature of the surroundings  $h_0$ , hence

#21:  $\frac{dh}{dt} = -k \cdot (h - h_0)$

where  $k$  is a positive constant. With "neutrale" designations we have

#22:  $\frac{dy}{dx} = -k \cdot (y - y_0)$

OR

#23:  $\frac{dy}{dx} = ky - by$

With  $a=k$  and  $b=ky_0$  we have

#24:  $\frac{dy}{dx} = ay - by$

To which the complete set of solutions are determined in the next paragraph b)

### 1. The level of the mathematician.

- To deepen the students' understanding of the rise from a situational to a referential model, questions at first level in should be asked.
- Obvious questions are:
  - (a) What is  $p$ ?
  - (b) What is  $k$ ?
  - (c) Can  $k$  be negative?
  - (d) What does  $p$  depend on?
  - (e) How can we solve equation #14 ( $dp/dt = kp$ )? etc.

### 2. The level of the deliberated mathematician.

The rise from referential model to general model in the case of growth gives the chance to discuss questions like:

- (a) 'What is  $y$ ?
- (b) What happened to  $p$ ?
- (c) Is  $k$  the same as it was before?
- (d) Is there any difference between #14 ( $dp/dt = kp$ ) and #20 ( $dy/dx = ky$ )? Which?
- (e) In what aspects are #14 and #20 similar?
- (f) What advantages could this change to neutral designations offer?
- (g) What are the possible drawbacks to this? etc.

The discussion leads to the next level of questions:

### 3. The level of the philosopher of mathematics

- Rise from general to formal model tends to happen over time. In the textbook, the repeated process of change to neutral designations may motivate a supportive debate
- The whole class can discuss questions like:
  - (a) What advantages could the change to neutral designations imply?
  - (b) What disadvantages?
  - (c) Is it always possible to choose neutral designations?
  - (d) What do we skip when we change?
  - (e) Is it possible to go back?
  - (f) What is  $y$  in #20 ( $dy/x = ky$ )?
  - (g) What does #20 describe or tell something about?

### 4. The level of the epistemologist

- Activities at the formal level may be widened by further reflections.
- The characteristics of mathematics and related issues can be enlightened by classroom discussions of questions like:
  - (a) Is it the same in other subjects – do they have neutral designations or the like?
  - (b) What kind of results can we get from this kind of procedures?
  - (c) Does it give a true picture?
  - (d) What does it mean that a model is true? etc.

Conclusion:

The use of ICT has potentials for a number of new aspects of learning. In particular, the use of ICT may leave more spare time in curriculum for modelling- and

multidisciplinary activities, compared to teaching without ICT. We see a recent tendency; content and focus of attention are changed, as a consequence of the reform's new requests and possibilities. My claim is that this change of focus can and should be directed by deliberated teaching of mathematical reflections, which may support a more balanced view on mathematics. The students' reflections should be raised by the teacher's preparation and use of a reflection guide, containing thought-provoking questions.

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