USE OF HANDHELD CALCULATORS IN MATHEMATICS EDUCATION

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Human tools are always situated in time and in a culture. This is also true for the tools used in the special sub-culture we call ‘school mathematics’. There are certain traditions to be considered, and changes in these are always bound to be met with suspicion and often with resistance. Traditional tools in mathematics education are for many paper-and-pencil, a ruler and a compass. But how ‘traditional’ are they, really? What about tools like various types of abaci, computational boards, slide rules or simply ones own fingers? All of these have been extensively used and are, in some cases, still used in different cultures (see for example Säljö, 2005).

When electronic equipment in the shape of ‘desk calculators’ became available in mathematics education in the mid 1970s, it was met by interest and a feeling of progression. But a number of questions were soon raised by teachers, researchers and others about the consequences for teaching and for students’ learning of mathematics. Among the negative arguments were that an extensive use of handheld calculators could harm students’ computational skills, both mental and by paper-and-pencil. On the other hand, many teachers could see inspiring possibilities with these new tools, especially those who tended to employ interactive or inquiry-oriented methodologies during instruction (Trouche, 2005a).

Early research of the use of calculators during the 1980s did not indicate that the apprehensions about their harmfulness were right. Researchers found that calculator use during instruction does not hold back either computational skills or conceptual understanding. Instead, it improves outcome results on non-calculator tests for all ability groups of students, as was shown in a meta-analysis of the use of calculators in schools mathematics by Hembree and Dessart (1992).

Scientific and programmable calculators appeared at upper secondary level in the early 1980s, and were rather uncontroversial. They were seen as handy tools replacing the extensive mathematical tables that were used before. Graphic calculators came in the late 1980s, and were at first considered mostly as visualising tools for calculus. But soon, teachers realised their potential for new approaches in most fields of school mathematics, such as number sense, algebra, geometry, data processing and statistics, analysis, etc. In fact, it became possible to organise instruction and to make assessments of students’ knowledge of mathematics in quite new ways. And it raised the pertinent question of what mathematical knowledge is and what type of skills students really should acquire during their school education.

Symbolic calculators (CAS) were introduced in the mid 1990s, and with them the questions about their use in school mathematics were intensified. Graphic calculators were by then widely used in instruction, and in many countries explicitly mentioned in curriculum (Trouche, 2005a). But the common use of CAS has for many reasons been delayed, e.g. because most teachers have got no training in how to handle them and to take advantage of their possibilities. At the same time, there can been institutional changes that force the use of CAS. One example of this is that symbolic calculators are partly allowed in all national tests at upper secondary level in Sweden since 2007.

The use of calculators is obviously not unproblematic, especially for the more advanced types like graphic and symbolic ones. Critical questions are constantly raised, both by those working within the school system and those that see it from the outside. Extensive research has also been made, covering a number of important aspects of the use of calculators in classrooms. The results of this research form a growing body of knowledge that can give answers to these critical questions.

Here, the results of research will not be presented or discussed. Instead, some of the most important questions in some areas will be formulated, accompanied with references to research literature that deal with these questions, partly or as a whole. All references are examples, and can not bee considered to be complete in any sense.

THEORETICAL FRAMEWORKS

In most literature concerning the use of calculators, the theoretical framework is more or less transparent. Some researchers deeply discuss this background, but some only provide rather shallow accounts of it. As a macro-theory stands, in most cases, Vygotskijs (1978) *socio-cultural learning*, and his concepts of *artefacts* and *mediating tools* (see also Säljö, 2005).

This theory has undergone a progression in different directions. One example of this is Wartofsky's (1979) classification of *primary, secondary and tertiary artefacts*, and another is *activity theory* (see Nardi, 1996) with the Agent-Objective-Others triad. Important are the concept of *instrument* and the process of *instrumental genesis*, which is referred to in much of recent literature. They were first introduced by Verillon and Rabardel (1995), and has since then been further developed and elaborated by others (Guin & Trouche, 1999; Artigue, 2002; Trouche, 2005a; 2005b). Instrumental genesis is the process of building of an instrument from an artefact, and has two closely interconnected components; *instrumentalisation*, directed toward the artefact, and *instrumentation*, directed toward the subject (the student).

The user's development of *instrumented action schemes* for working with various tasks is of vital importance (Guin & Trouche, 1999; Drijvers, 2002a, Drijvers & Gravemeijer, 2005), and also the distinction between between *instrument* (the artefact is used in utilisation schemes) and *instrument for* *semiotic mediation* (the teacher uses it to develop a specific meaning related to the mathematical content) (Radford, 2001; Mariotti, 2002).

A basic question in the learning of mathematics is which cognitive systems that are required to give access to mathematical objects. Duval (2006) has created a framework of *representational registers* that are mobilized in mathematical processes, and he distinguishes between two types of transformation of semiotic representations, *treatment* and *conversion*. In all these transformation processes calculators can play an important role, with their possibilities to simultaneous representations and manipulations of mathematical objects.

RESEARCH AREAS AND QUESTIONS:

Effects on students' skills and achievements

- To what extent and for which purposes are calculators used in mathematics instruction? (Jacobsson, 2001)
- How are they used in tests and national exams? (Scheuneman et al, 2002; Graham et al, 2003; Ellington, 2003)
- What impact has the use of calculators on the results of these tests and exams? (Scheuneman et al, 2002; Heller et al, 2005; Antonijevic, 2007)
- Are there positive/negative effects on students' capability to work out tasks by paper and pencil or by mental calculation? (Ellington, 2003)

Specific areas of mathematical knowledge

- Reports of local experiments, e.g. introducing quadratic functions and equations (observations).
- How does the use of calculators affect students' understanding and use of mathematical symbols?
- What are the effects on students' number sense and the understanding and use of operators?
- Algebraic rules (Kieran & Drijvers, 2006) and literal symbols (hidden number, generalised number, variable, parameter) (Graham & Thomas, 2000; Drijvers, 2003; Drijvers & Gravemeijer, 2005)?
- Functions (Bardini et al, 2004; Rivera et al, 2004) and calculus? (Lagrange, 1999)
- Statistics (Forster, 2006)? Geometry? Analysis? etc.

Teaching and learning, practice ↔ theory

- Which changes in teaching practice are needed when different calculators are used? (Heid, 1997; Heid & Edwards, 2001; Ellington, 2003)
- In introducing and investigating new concepts? (Yerushalmy, 2006)
- In modelling and problem-solving? (Drijvers, 2002a)
- In visualising and changing of representational forms? (Lagrange, 1999)

- With justifications and proofs? (Kieran & Drijvers, 2006)
- How do students make records of their work with calculators as tools? (Ball & Stacey, 2005)
- How do students connect results of their calculator work with mathematical theory? The teachers role. (Drijvers, 2002a; Kieran & Drijvers, 2006)
- How do the various types of constraints of calculators shape students' perception of mathematical activity and concepts? (Trouche, 2005b)
- In what way have textbook changed to in order to adapt to the demands of calculator using in school mathematics? (Trouche, 2005a)
- What kind of didactical engineering is needed to integrate calculators into education? (Artigue, 2005)

Socio-cultural learning and activities

- How do students cooperate and communicate when they work with tasks, using calculators? (Radford et al, 2003; Forster & Taylor, 2003; Rivera & Becker, 2004; Thomas & Hong, 2004)
- Which types of instrumentation schemes and working styles can be observed when students work with calculators? (Drijvers, 2002a; Drijvers, 2003; Berry & Graham, 2005; Berry et al, 2006)
- Can gender differences be observed in using calculators? (Burrill et al, 2002)
- What are the effects of calculator-using for lower-achieving students and those with learning problems? (Hennessy et al, 2001; Steele, 2007; Yerushalmy, 2006)
- What are the possibilities? (Steele, 2006)

Affective factors, obstacles and students' beliefs.

- Which beliefs do students have concerning the use of calculators and their own learning of mathematics? (Ellington, 2003; Pierce & Stacey, 2004; Reznichenko, 2007)
- Which obstacles do students encounter, using calculators? (Drijvers, 2000a; 2002b; Ruthven, 2002; Kieran & Saldanha, 2005)
- How are the affective factors (motivation, joy, satisfaction etc.) influenced for the students? (Pierce & Stacey, 2004; Reznichenko, 2007)

Teachers' competence and beliefs.

- What are the beliefs and intentions of teachers using calculators in their practice? (Kendal & Stacey, 2002; Thomas & Hong, 2005; Tan & Forgasz, 2006; Brown et al, 2007)
- Which types of in-service training and support would be desired by teachers for developing their competence in using calculators in their teaching? (Ball, 2004)

HANDHELD CALCULATORS IN THE FUTURE

What prospects do calculators then have in the future? Computers in the form of laptops are slowly entering our classrooms, and there are those who claim that this more universal tool should be used instead of calculators, and that these soon will be obsolete. So are they worth the time, effort and money to invest in for teachers as well as for schools?

On the other hand, another handheld device exists that successfully has taken over several functions, namely the mobile phone. The newest generation has even the capacity to replace computers in many respects. Most students are quite experienced in taking advantage of the mobile phone's possibilities, and in a few years the 'handheld calculator' could very well be a software that you download to your phone. In fact, development projects in that direction already exist, e.g. at Malmö University.

A clear tendency is that at least more advanced calculators and computer software seem to converge. Features and interface of the newest generation of calculators are more 'computerlike', and they often come with software that connects them with the computer or even emulates them (e.g. Texas T-Nspire). So the borderline between calculators and computers, and also to other ICT-tools, has become more difficult to draw. It might be futile to discuss which of the two that is 'best'. Instead maybe we ought to see both calculators and computers as valuable parts of an ICT-system that we use as a general tool for enhancing students' learning of mathematics.

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