# Use of ICT in school mathematics

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# Change of content linked with the use of ICT/CAS in upper secondary mathematics

The use of ICT/CAS is commonly linked with teaching that favours

- Problem solving
- Numerical modelling and solution and qualitative illustrations
- Technical aspects

On the expense of proof- and theory issues

# Results from a reseach project, example

Learning differential equations with the use of 'Derive'

- Numerical methods for solving Differential Equations (DE) provides for mathematical modelling released from traditional restrictions in the form of request for analytical solution
- Exploration of DE models supports the students formation of conceptions like solution, families of solutions, equilibrium point etc.
- The interpretation in a suitable notion of the students' explorative work serves to to point out crucial elements of their modelling activities, which are facilitated by the use of computer routines.

# Recent developments in upper secondary school mathematics in Denmark

- The use of CAS in parts of the written examination was prescribed
- A number of multi disciplinary projects and activities were introduced
- New forms of writing were introduced in mathematics: reports, synopsis etc

# The new upper secondary:

After the reform of upper secondary, we have in mathematics:

- Teachers with a high degre of professional autonomy
- Rather detailed syllabus corresponding to the written exaimination
- Spare time for optional themes, multidisciplinary projects etc (app. 25% of the time)

# Change of content, linked with multidisciplinary activity

The mathematical content of reports and dokuments from the multidisciplinary activities tends to be:

- Tools for graphic illustration, calculations and/or statistics
- About mathematics histrory, philosophy, culture etc.

These issues were seldom dealt with, before the reform.

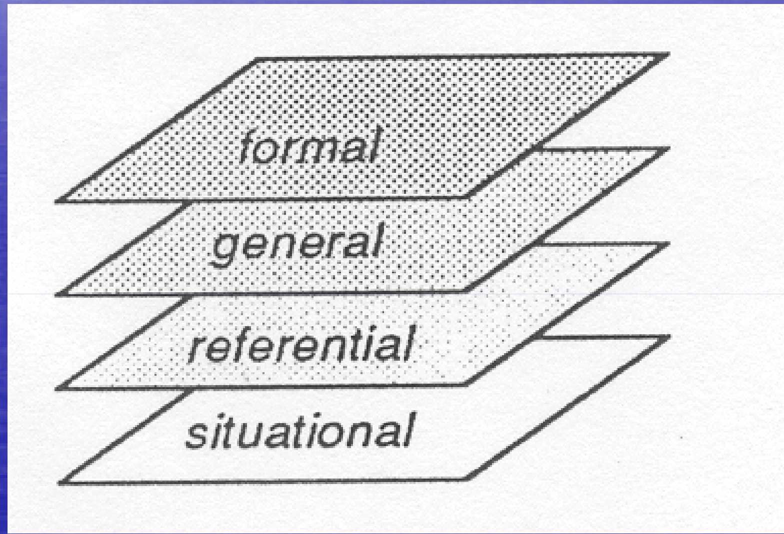
# Two aspects of changes:

- Increased focus of attention on mathematics as a tool for calculation, estimation and concrete modelling and problem solving
- Broadening out the field of study within school mathematics

What about mathematics ’itself’?

# Combination of two models for learning and reflection.

Each rise from one level to the next in the model of mathematical activity can be supported by reflections at a corresponding level.




*Levels of activity.*
*Gravemeijer, K. & Stephan, M. (2002). p.159*

# Stronger focus on mathematical reflections:

- To support the content of learning and doing mathematics
- To develop the students’ mathematical competencies and profound understanding of mathematical activity and theory
- To strengthen the students’ ideas of what mathematics is about

# About mathematical reflections

- Reflections presupposes and supports the development of intellectual autonomy
- Reflections cannot be imposed on the student – they must take their free course
- Reflections may be guided by the teacher, be inspired by thought-provoking questions

# Preparation and use of a reflection guide

- Helps the teacher to be concious about and to use thought-provoking questions about mathematical thinking
- Aims to structure the teacher's analysis of students' learning trajectory
- Helps to give weigth (by the students and the teacher) to the core perspective of mathematics as an activity and, thereby, to give a more balanced wiew on mathematics

# Preparation of a reflection guide, guidelines

- Choice of a suitable part of the students' imagined learning trajectory
- Design of the structure of inclusion in the teaching sequence
- Identification of potential levels of activity in the sequence and/or in the materials
- Formulation of thought-provoking questions

# Preparation of reflection guide, exmple

- ENCL III is an excerpt from a booklet on differential equations from a modelling perpective.
- The example is based on this excerpt

**ENCL III:** Let us now try to solve the differential equation

#14: $\frac{dp}{dt} = kp$

As a model for unbounded growth of a population with k = 0.04 and P(0)= 4.6 , we obtain

#15: DSOLVE1(0.04·p, −1, t, p, 0, 4.6)

Which gives

#16: $25 \cdot LN(p) - t = 25 \cdot LN\left(\frac{23}{5}\right)$

And therefore

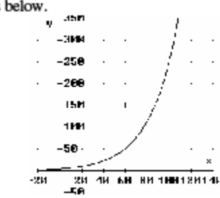
#17: $SOLVE\left(25 \cdot LN(p) - t = 25 \cdot LN\left(\frac{23}{5}\right), p\right)$

#18: $p = \frac{23 \cdot \hat{e}^{t/25}}{5}$

or

#19: $p = 4.6 \cdot \hat{e}^{0.04 \cdot t}$

Hence, we have exponential growth. The graph is below.




It is common to use "neutral" designations like x for the independent and y for the dependent variable. The differential equation that describes the unbounded growth of a population then turns to:

#20: $\frac{dy}{dx} = ky$

To which the complete set of solutions are determined in the next paragraph a)

The temperature of coffee in a cup changes over time and experiments have shown that the rate of change of the temperature h is proportional with the difference between h and the temperature of the surroundings h0, hence

#21: $\frac{dh}{dt} = -k \cdot (h - h_0)$

where k is a positive constant. With "neutrale" designations we have

#22: $\frac{dy}{dx} = -k \cdot (y - y_0)$

or

#23: $\frac{dy}{dx} = ky_0 - ky$

With a=k and b=ky0 we have

#24: $\frac{dy}{dx} = b - ay$

To which the complete set of solutions are determined in the next paragraph b)

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# *1.The level of the mathematician.*

- To deepen the students' understanding of the rise from a situational to a referential model, questions at first level in should be asked.
- Obviuos questions are:
  - *What is p?*
  - *What is k?*
  - *Can k be negative?*
  - *What does p depend on?*
  - *How can we solve equation #14 (dp/dt =kp)? etc.*

# 2.The level of the deliberated mathematician.

- The rise from referential model to general model in the case of growth gives the chance to discuss questions like:
  - 'What is y?
  - What happened to p?
  - Is k the same as it was before?
  - Is there any difference between #14 (dp/dt=kp) and #20 (dy/dx=ky)? Which?
  - In what aspects are #14 and #20 similar?
  - What advantages could this change to neutral designations offer?
  - What are the possible drawbacks to this? etc.
- The discussion leads to the next level of questions:

# *3.The level of the philosopher of mathematics*

- Rise from general to formal model tends to happen over time. In the textbook, the repeated process of change to neutral designations may motivate a supportive debate
- The whole class can discuss questions like:
  - *What advantages could the change to neutral designations imply?*
  - *What disadvantages?*
  - *Is it always possible to choose neutral designations?*
  - *What do we skip when we change?*
  - *Is it possible to go back?*
  - *What is y in #20 (dy/x=ky)?*
  - *What does #20 describe or tell something about?*

# *4.The level of the epistemologist*

- Activities at the formal level may be widened by further reflections.
- The characteristics of mathematics and related issues can be enlightened by classroom discussions of questions like:
  - *Is it the same in other subjects – do they have neutral designations or the like?'*
  - *What kind of results can we get from this kind of procedures?*
  - *Does it give a true picture?*
  - *What does it mean that a model is true?* etc.

# Conclusion

- The use of ICT has potentials for new aspects of learning
- Gives room for other, modeling- and multidisciplinary activities
- Content and focus of attention are commonly changed, consequently
- Mathematical reflections may support a more balanced view on mathematics
- Reflections should be raised by thought-provoking questions